# QUEUEING MODELS FOR 4D AIRCRAFT OPERATIONS

Tasos Nikoleris and Mark Hansen

EIWAC 2010

# Outline

- Introduction
- Model Formulation
- Metering Case
- Ongoing Research

# Time-based Operations

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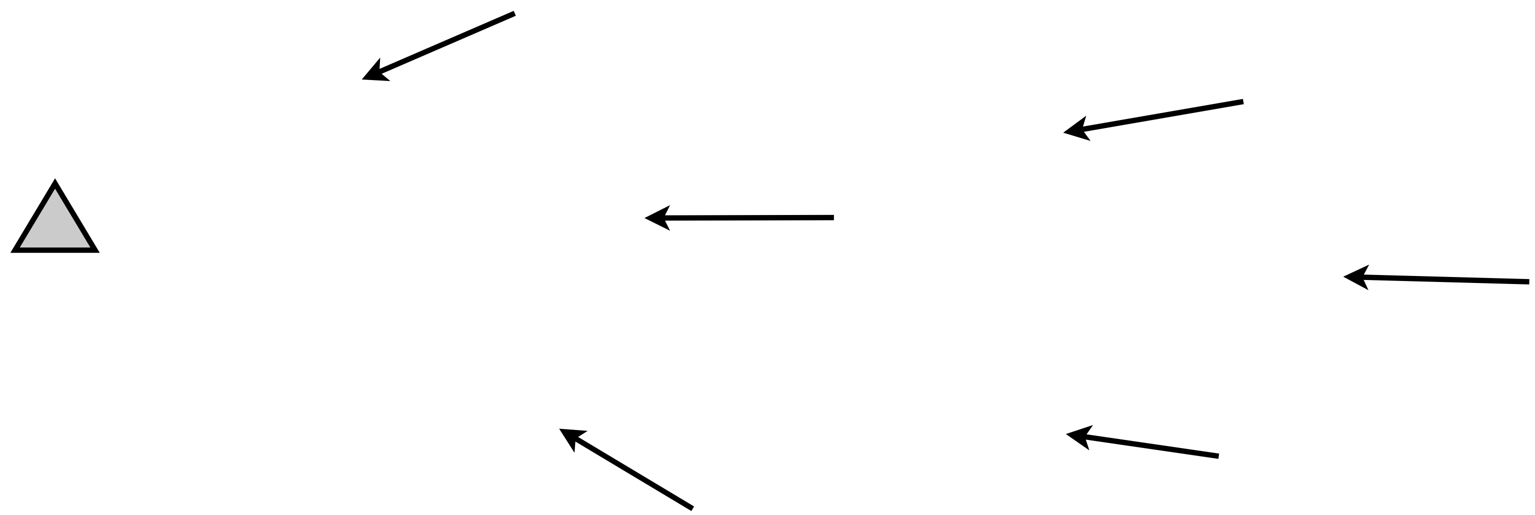

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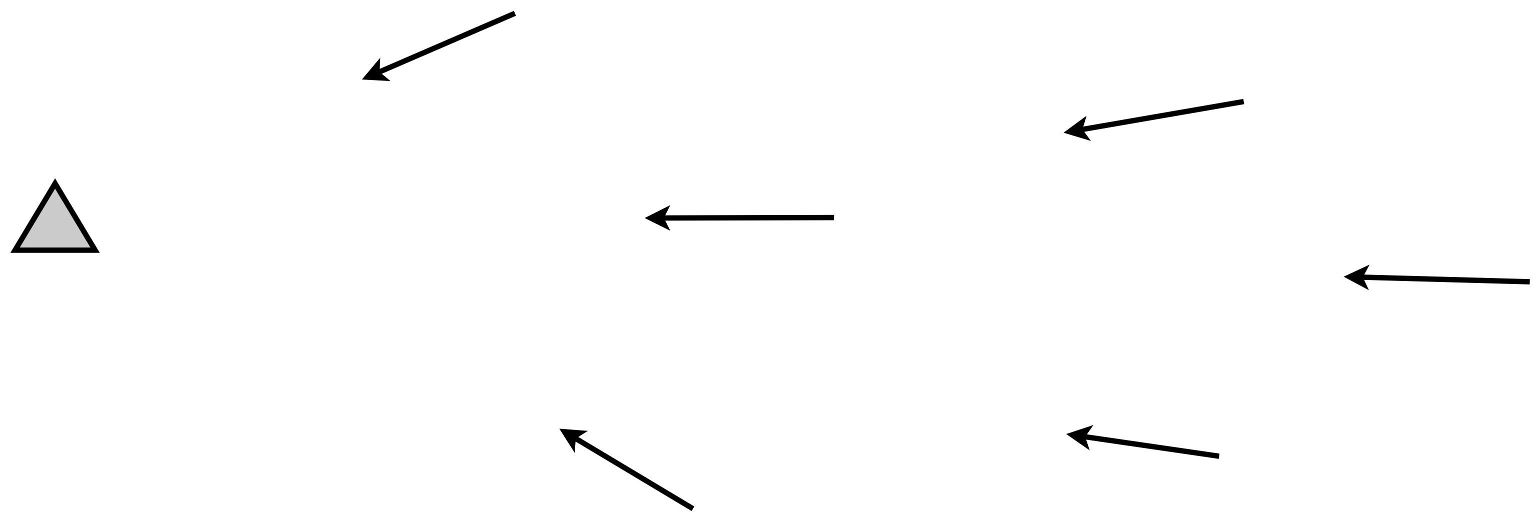

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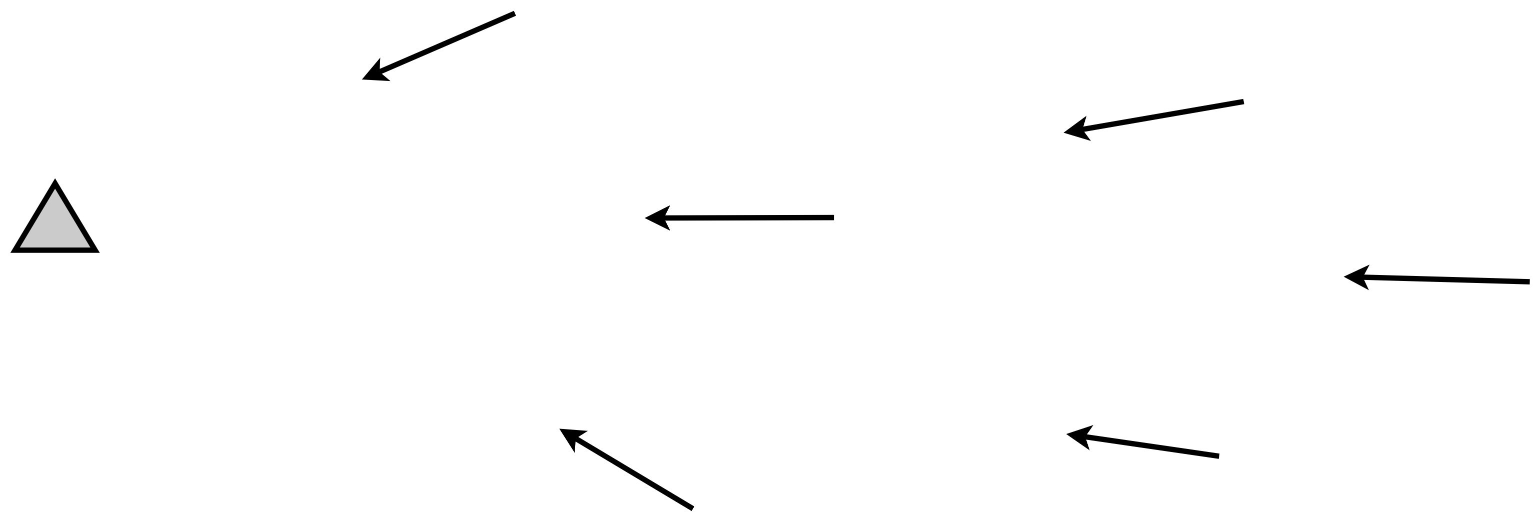

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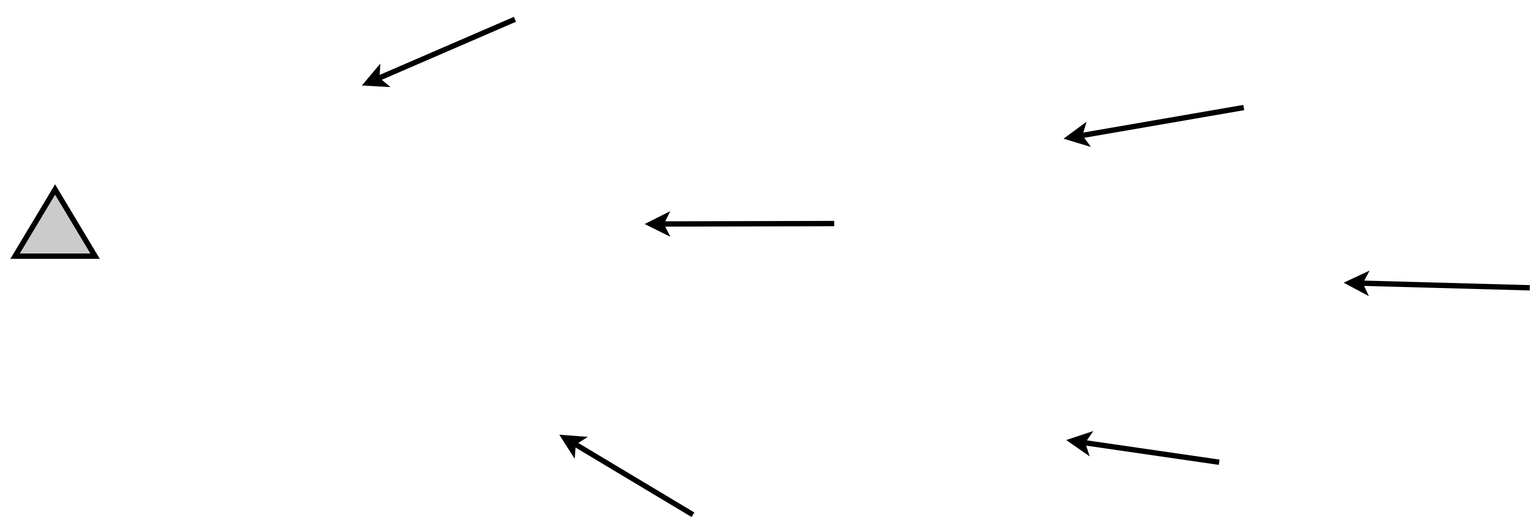

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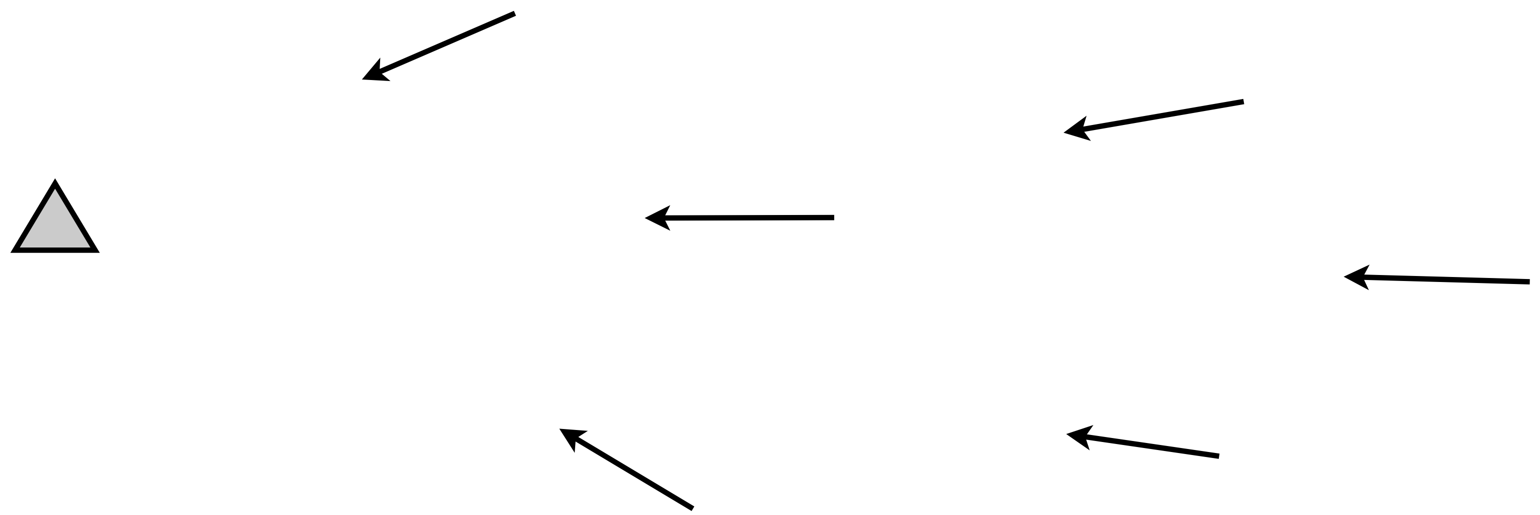

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➡ **Delay to traverse the fix as function of precision ?**

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- Estimate **queueing** delay for each aircraft to cross that fix

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- Aircraft meet RTA's with some stochastic lateness (±)

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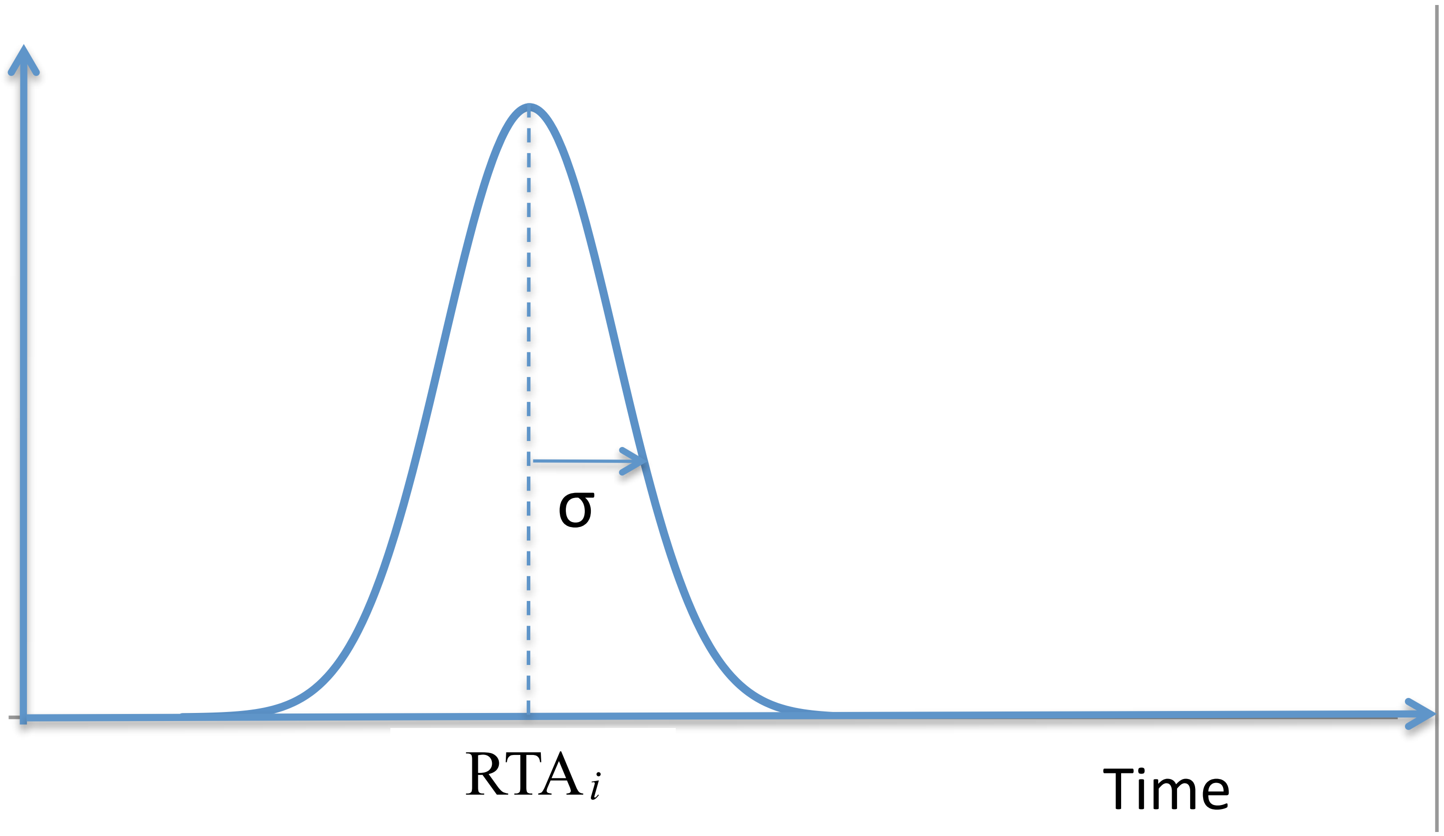

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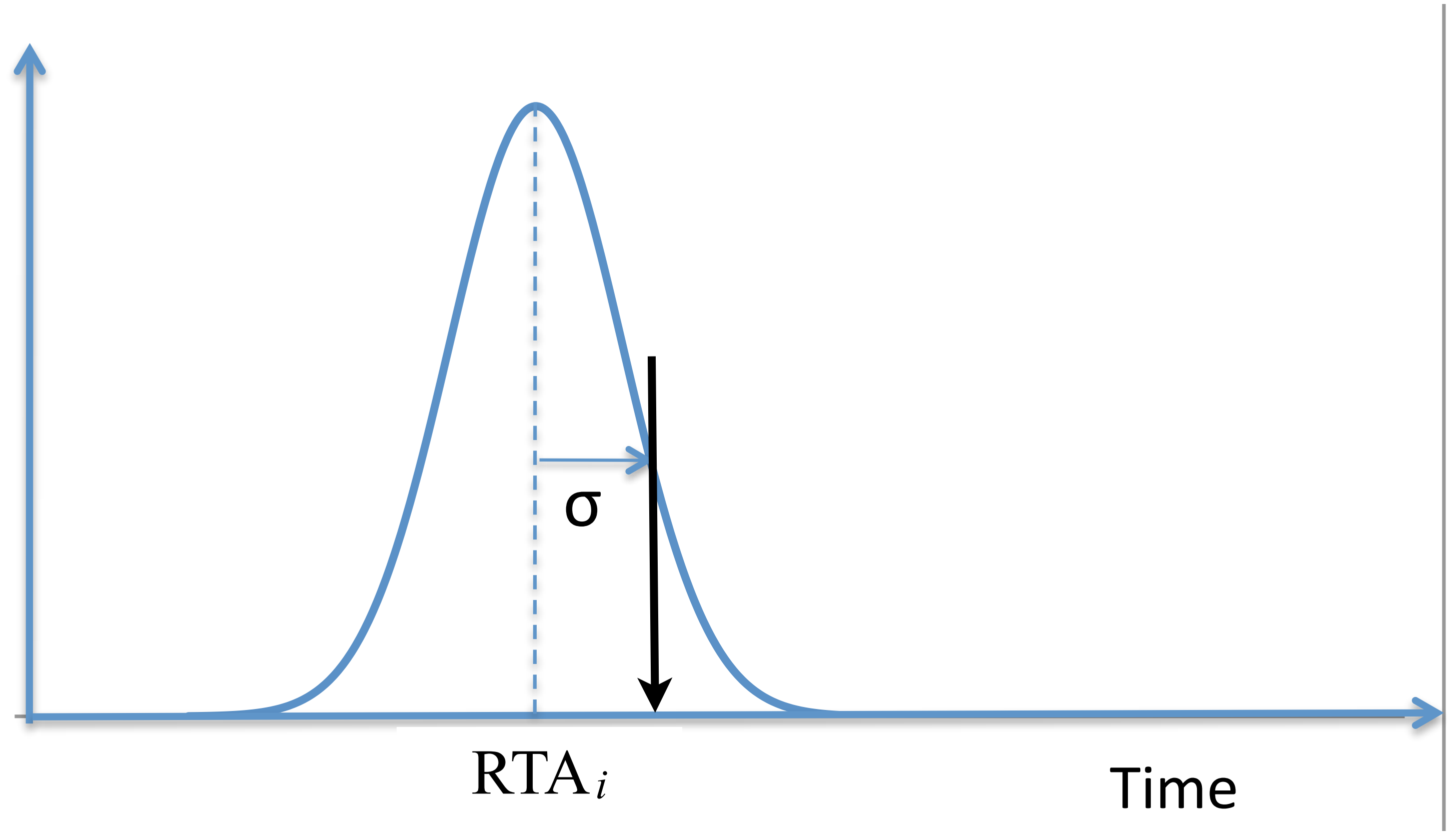

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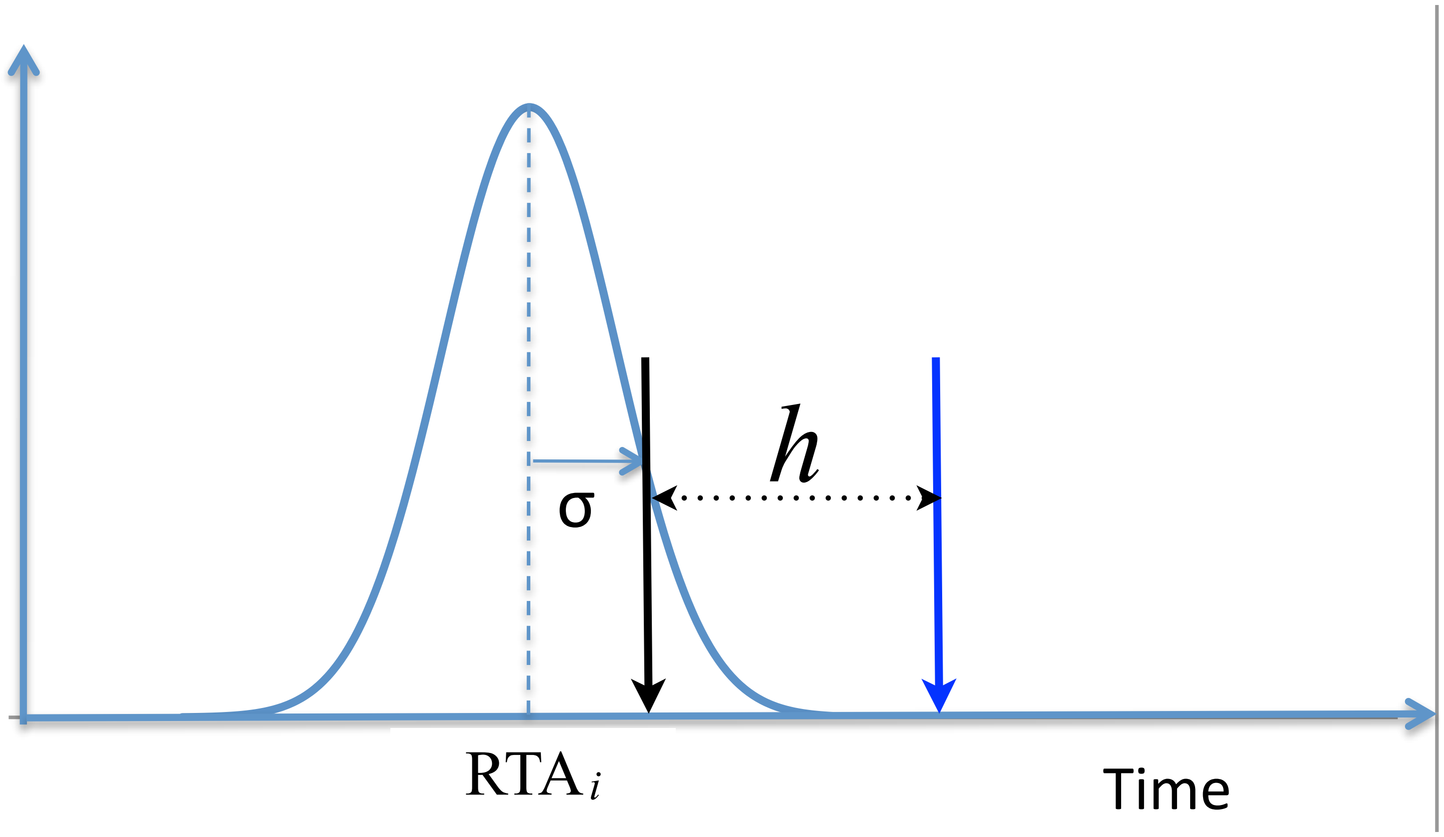

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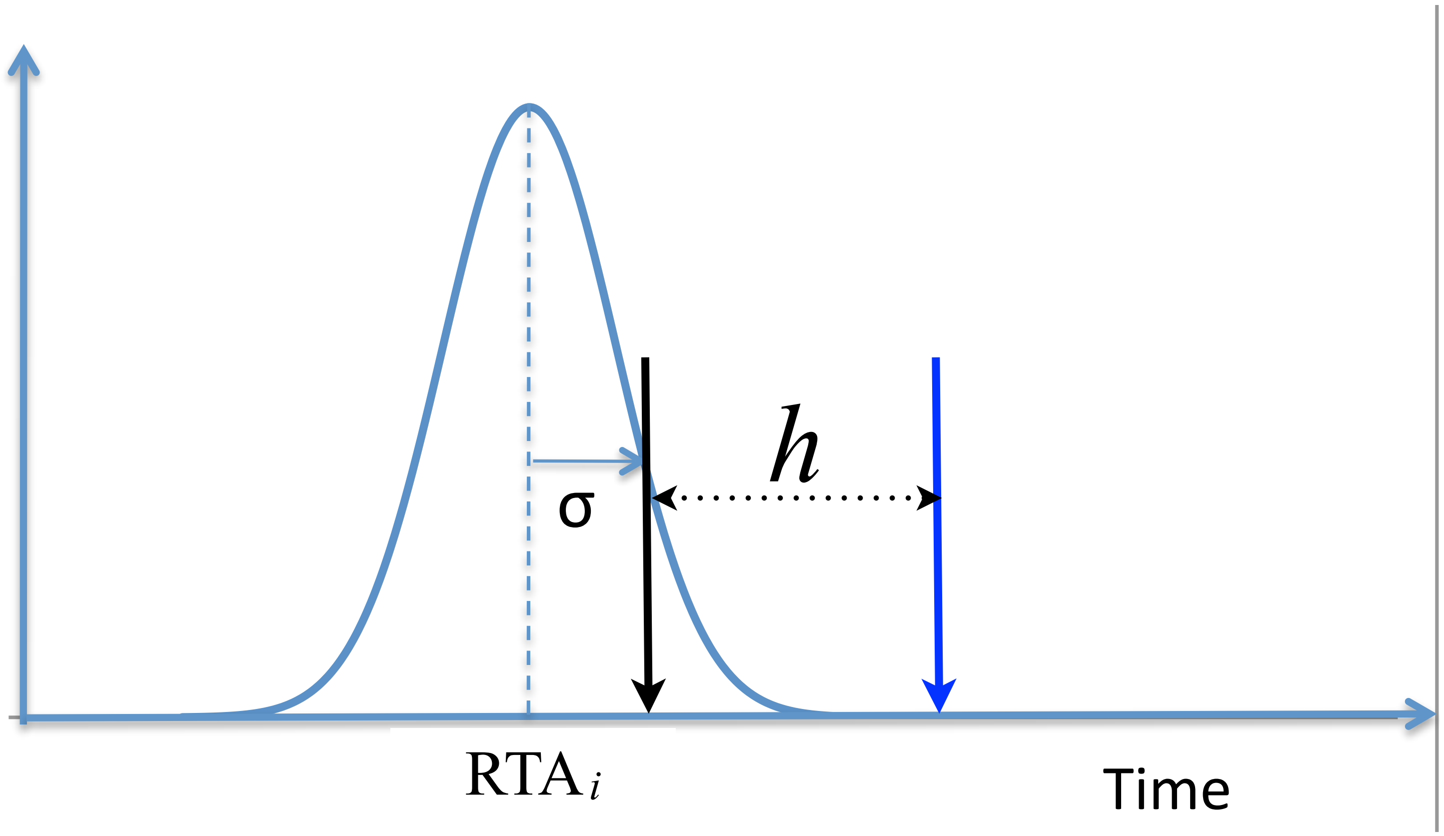


- First-Scheduled-First-Served (no overtakings)

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- Clark (1961)
  - derives mean and variance of $max(X, Y)$
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- Use Clark Approximation Method recursively to estimate $\mathrm{E}[D_i]$ and $Var[D_i]$

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- Compared estimates of the Clark method with average of $10^4$ Monte Carlo simulation runs

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- Total Loss = Deterministic + β * Stochastic

# Queueing diagram example

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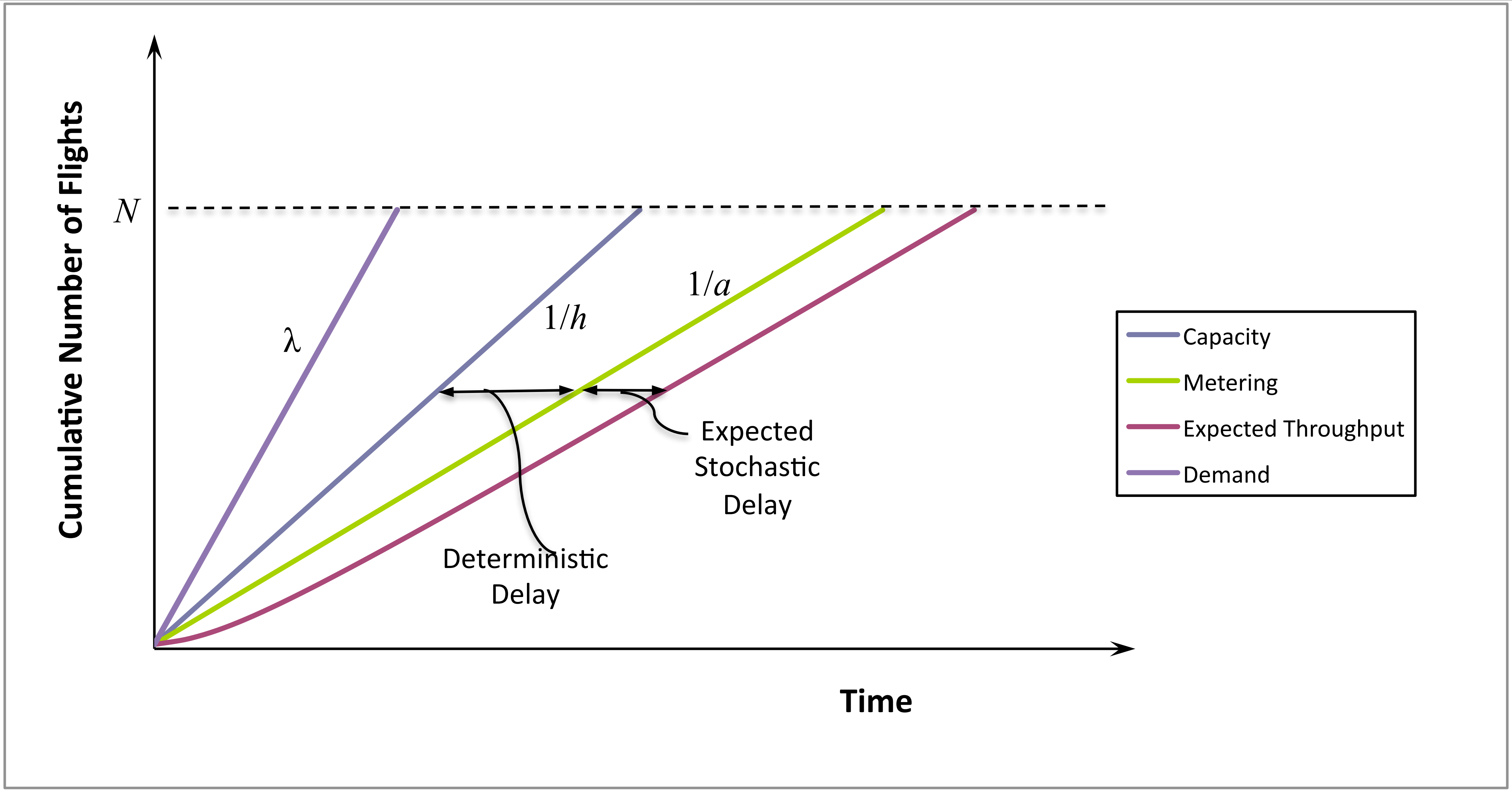

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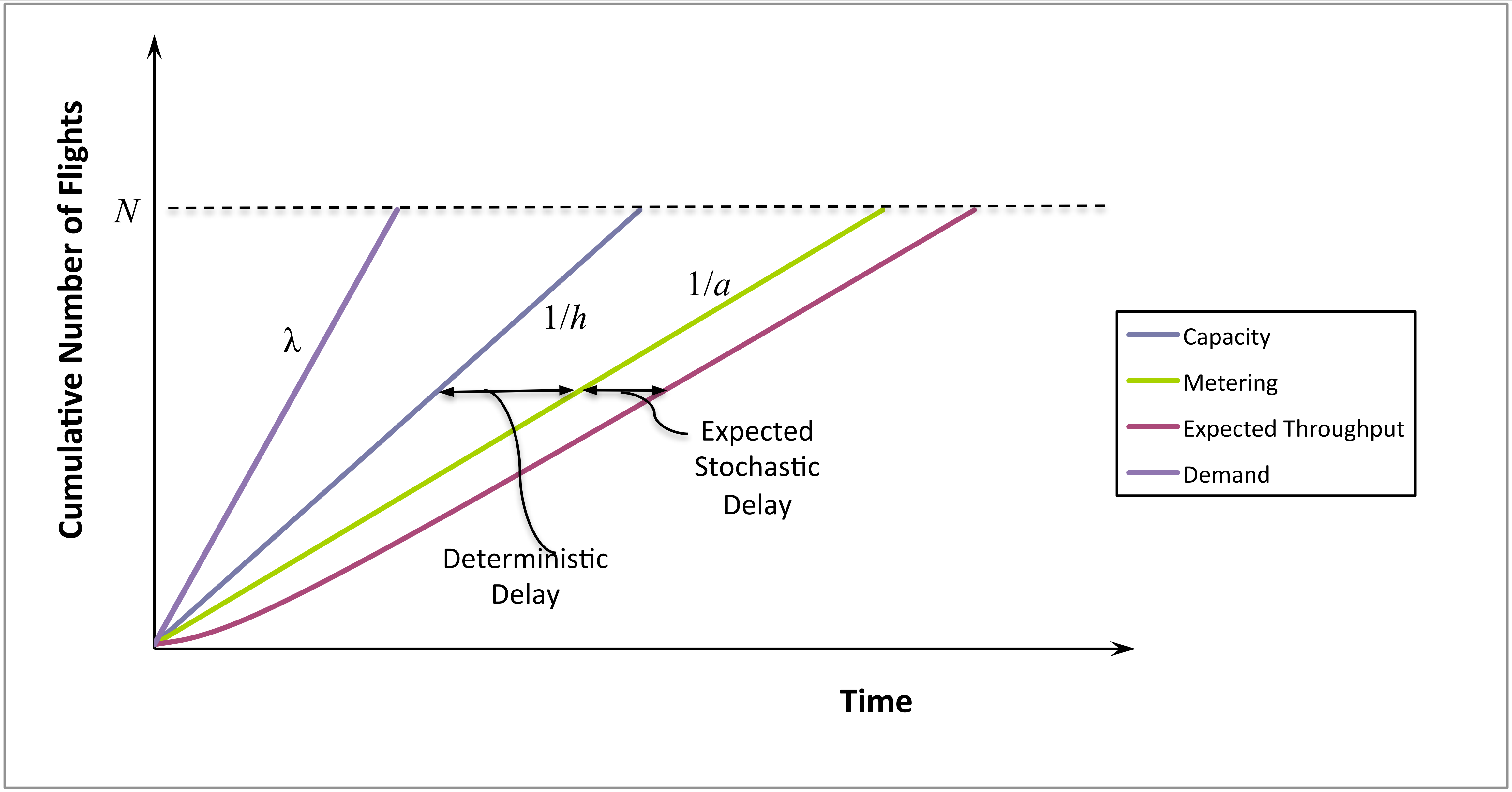


- Deterministic ~ $N^2$, Stochastic ~ N

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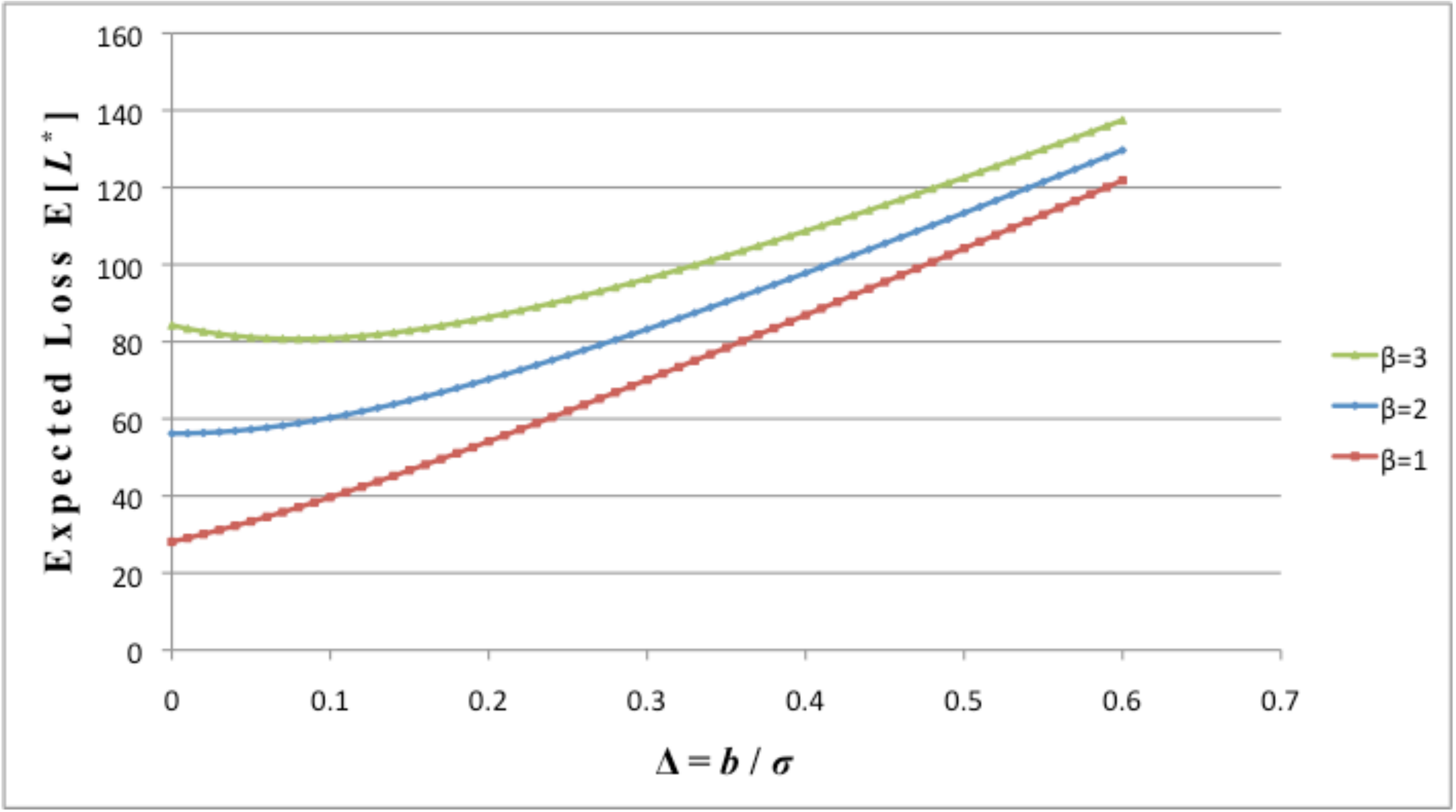

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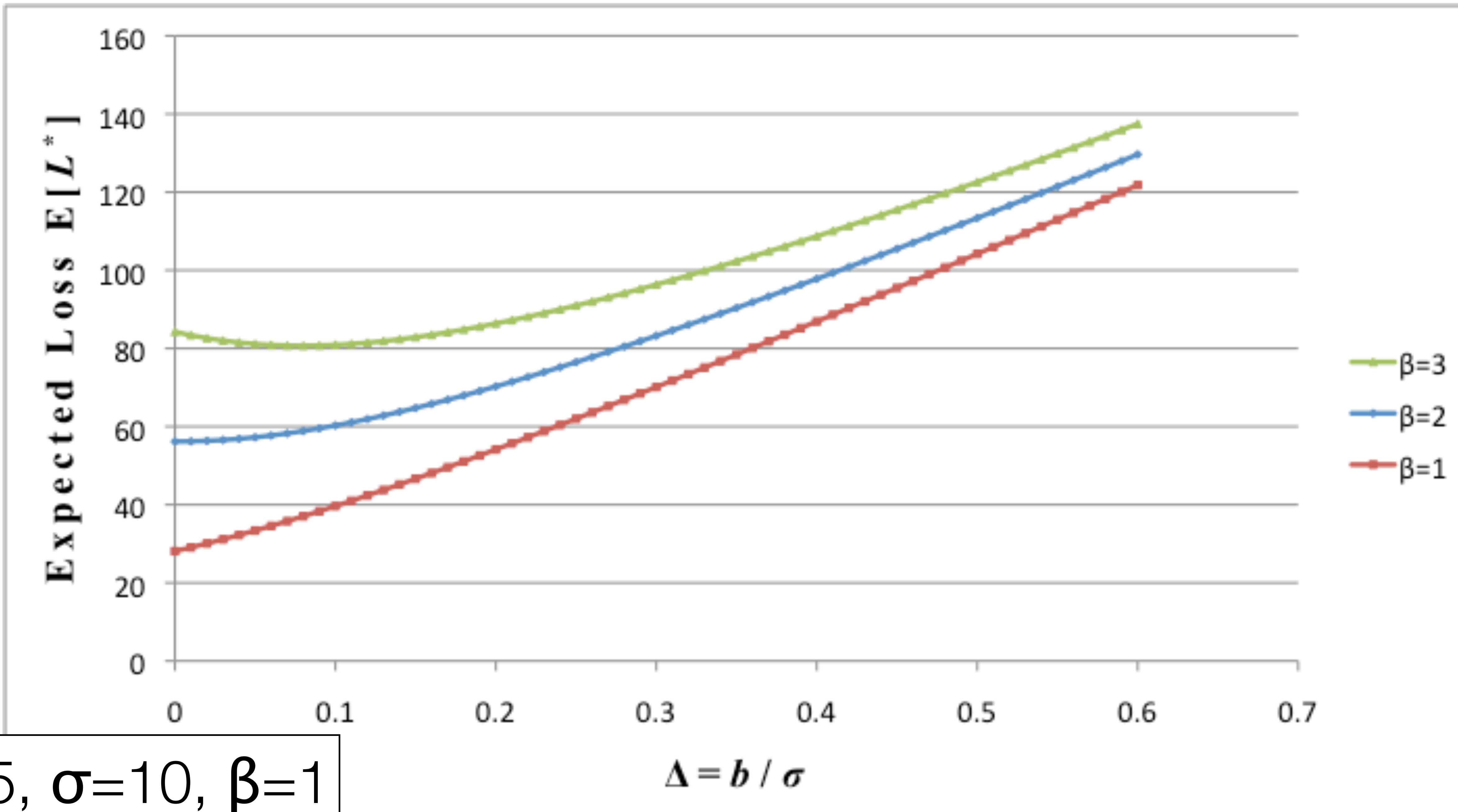


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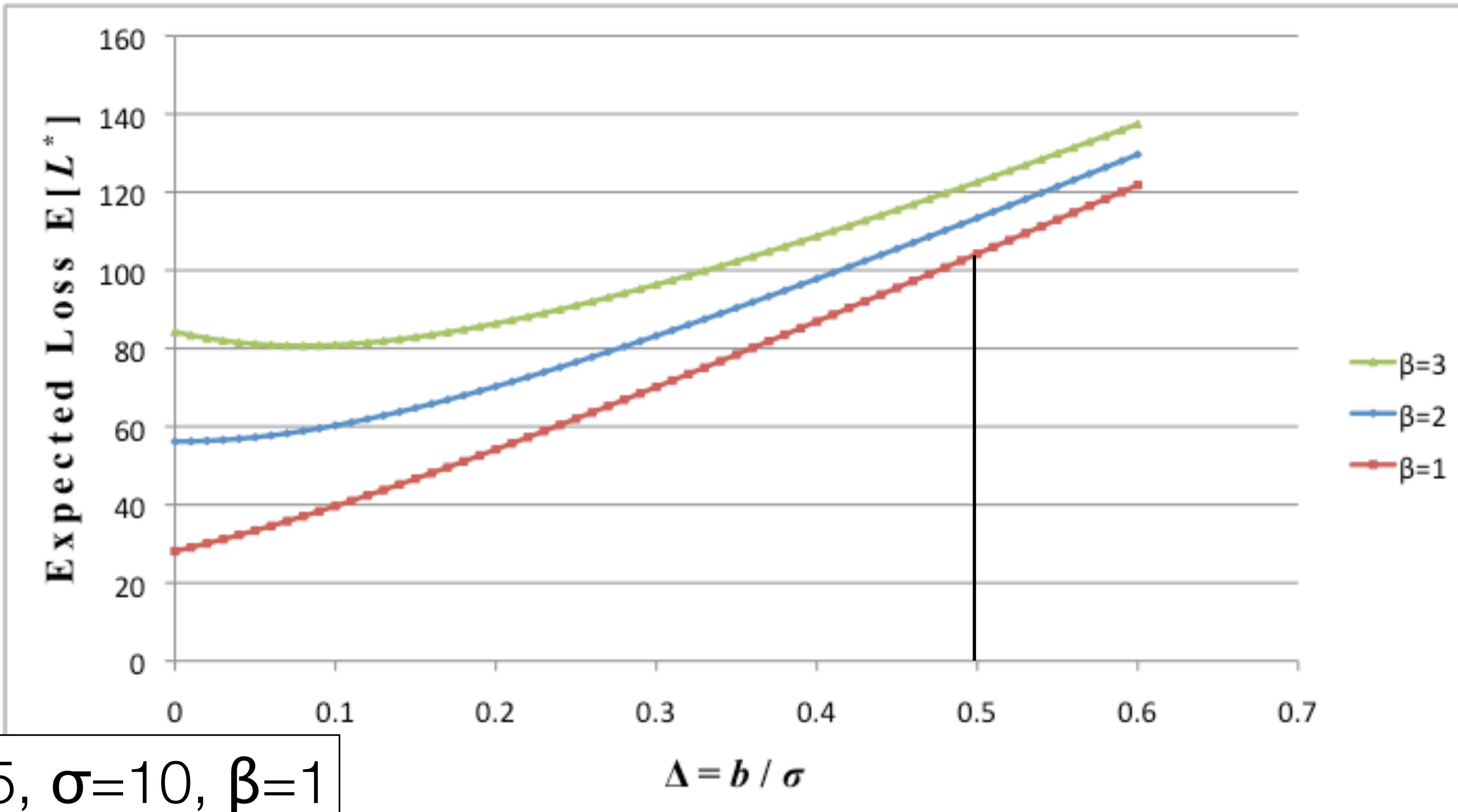


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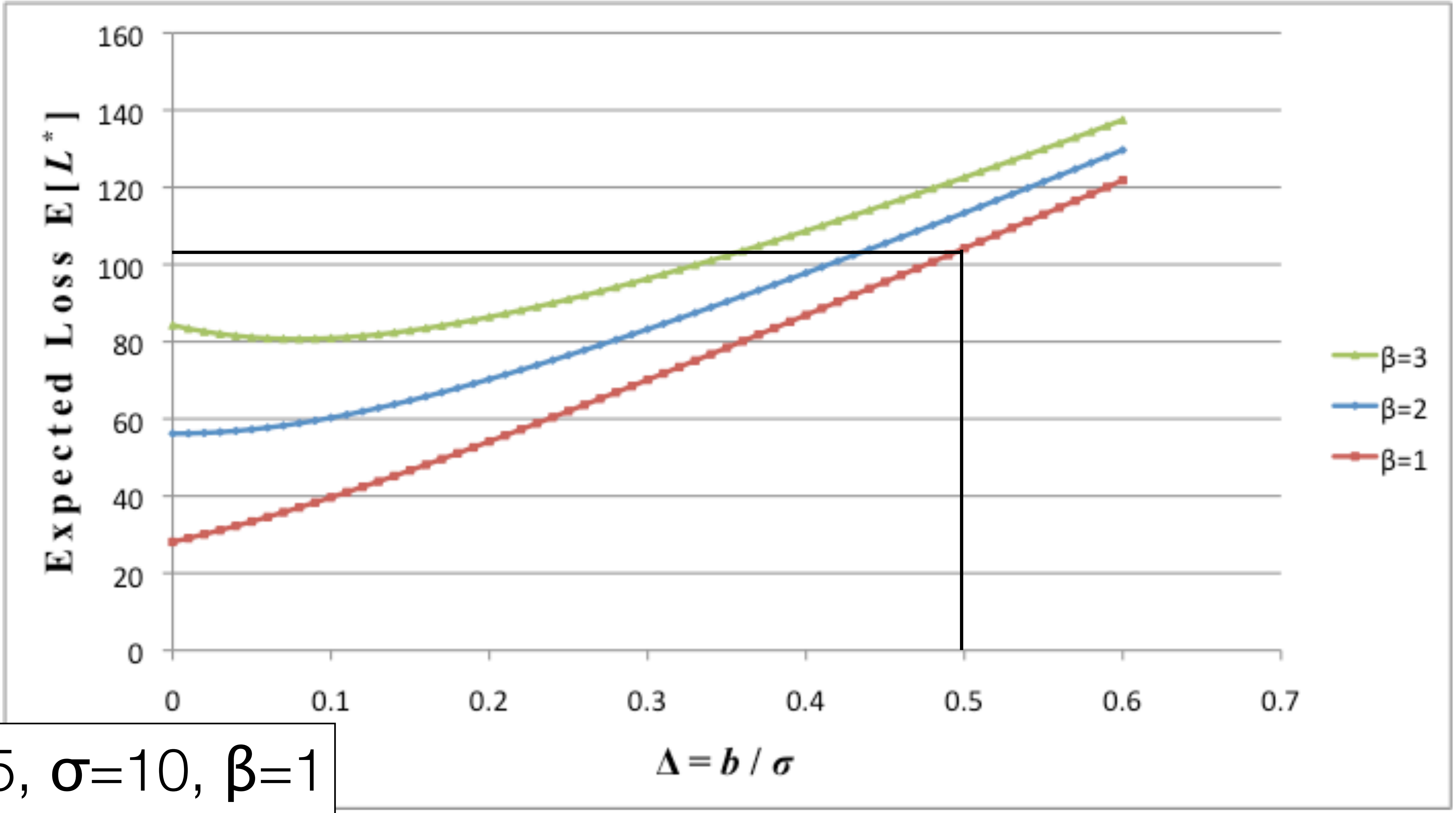


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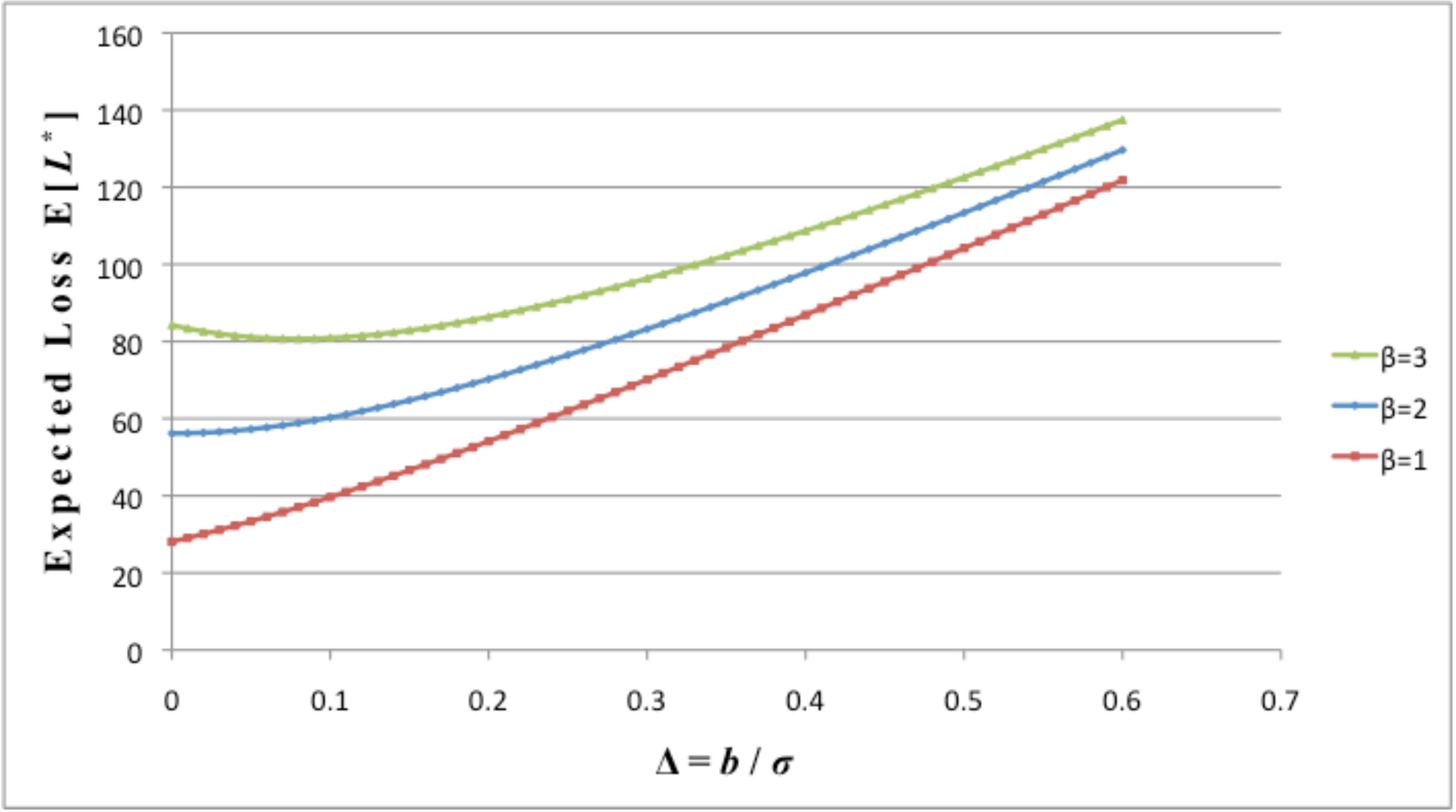

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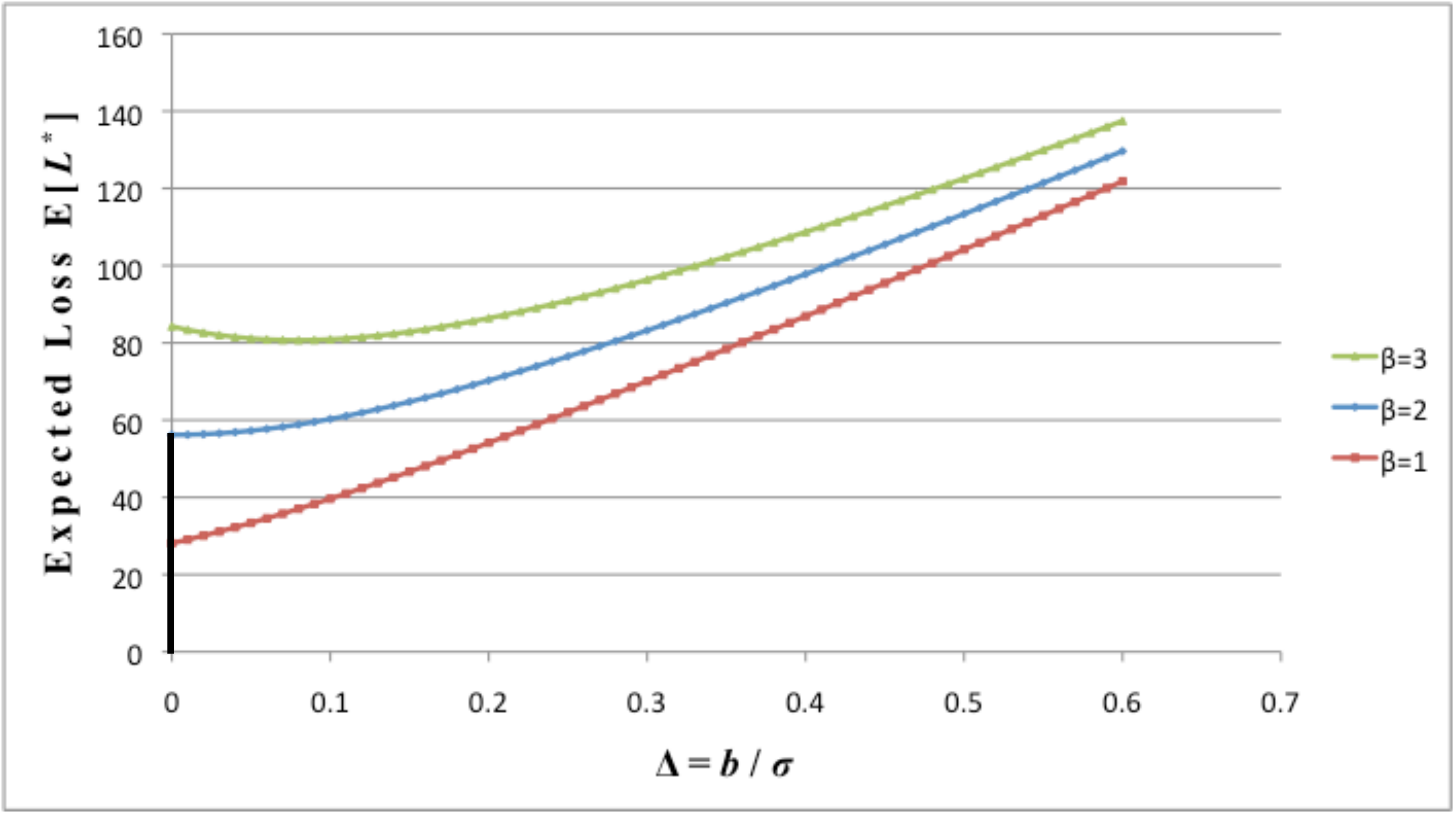

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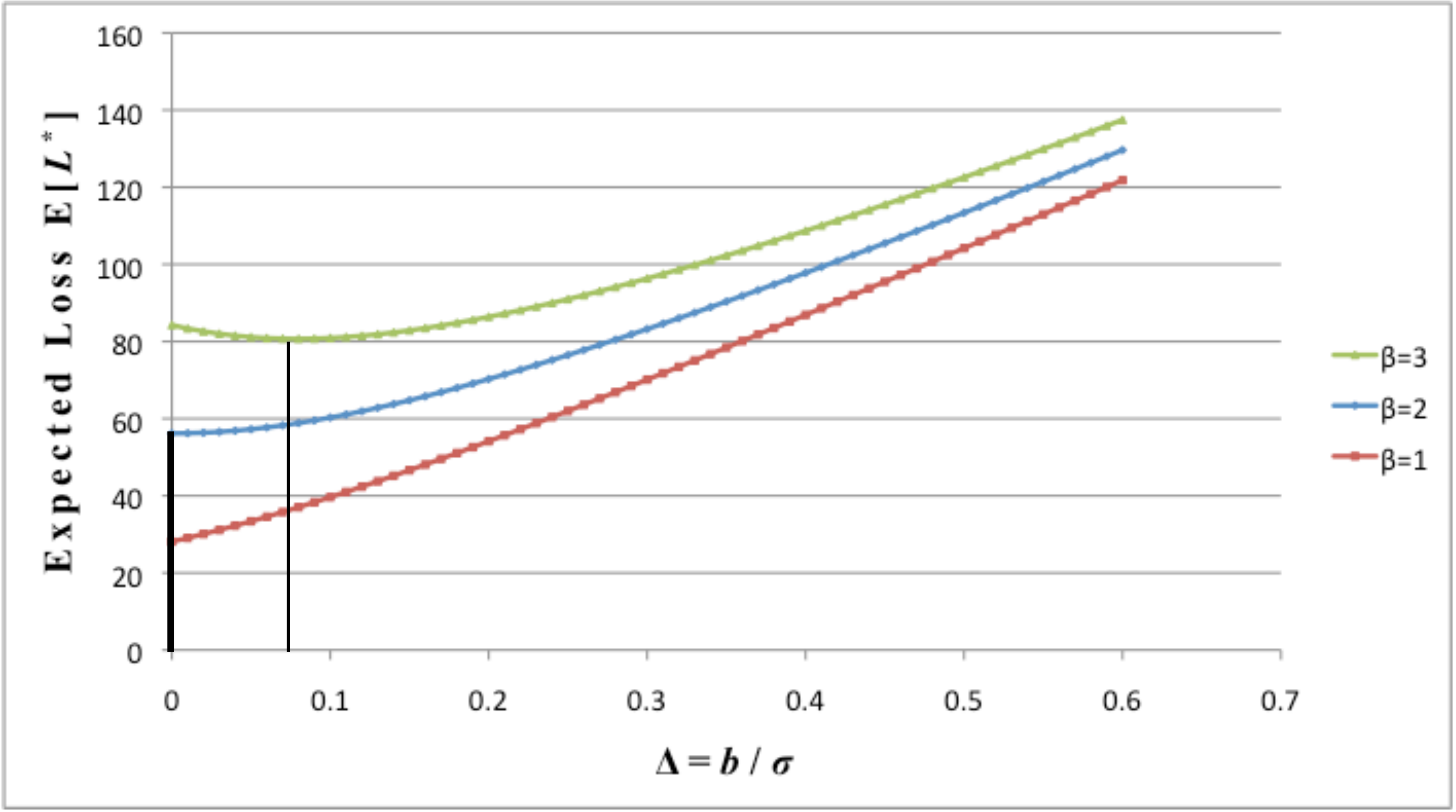

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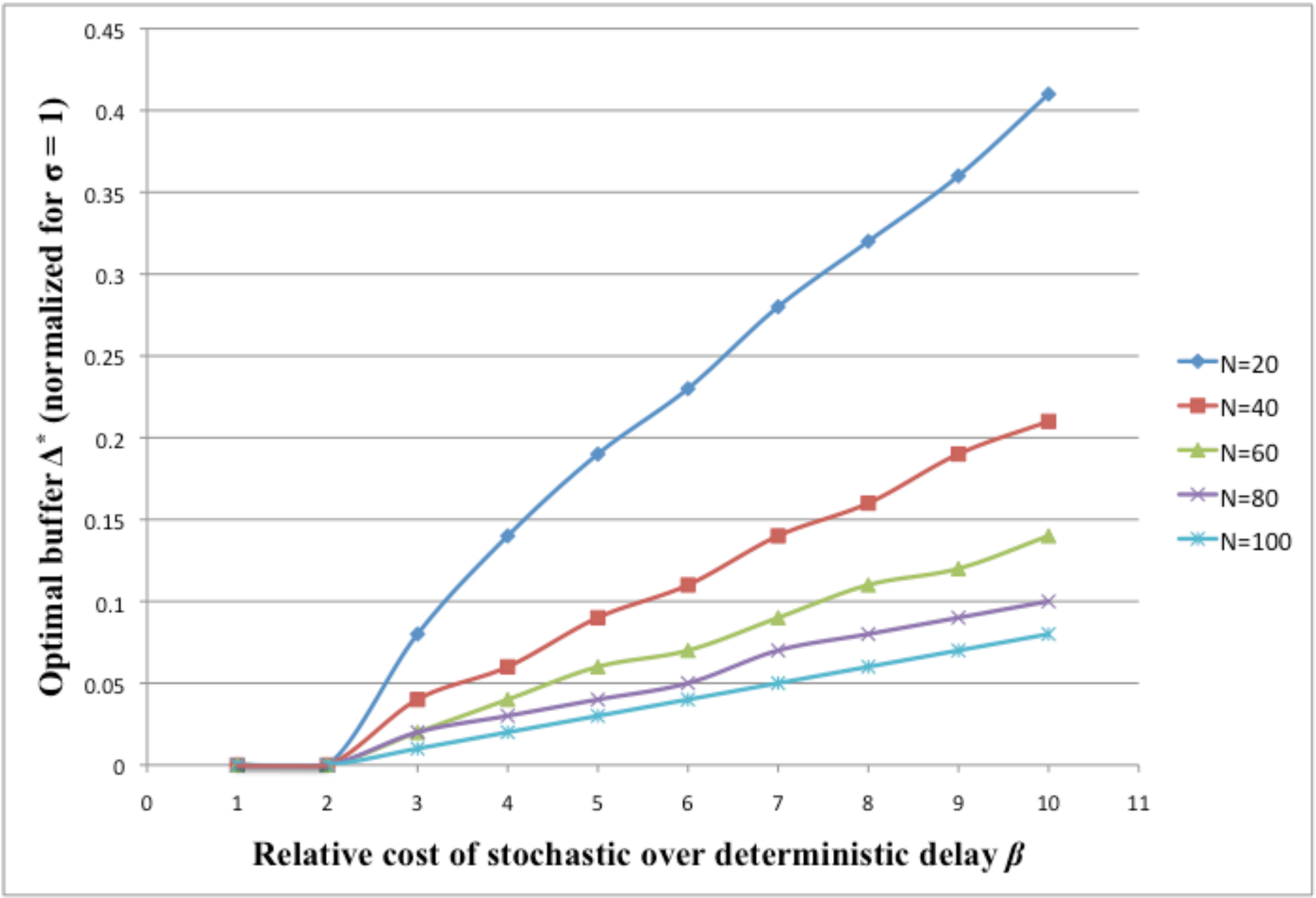

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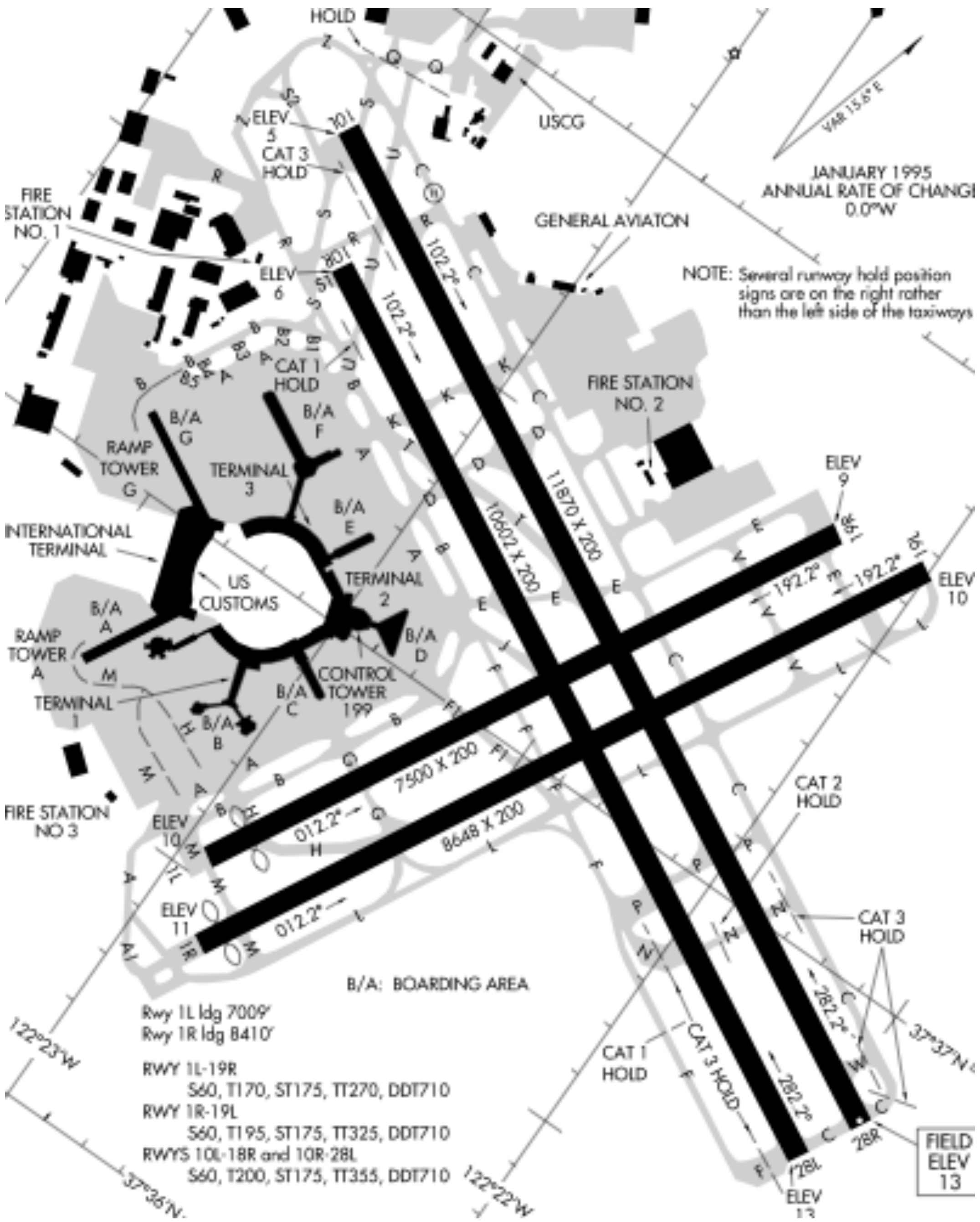

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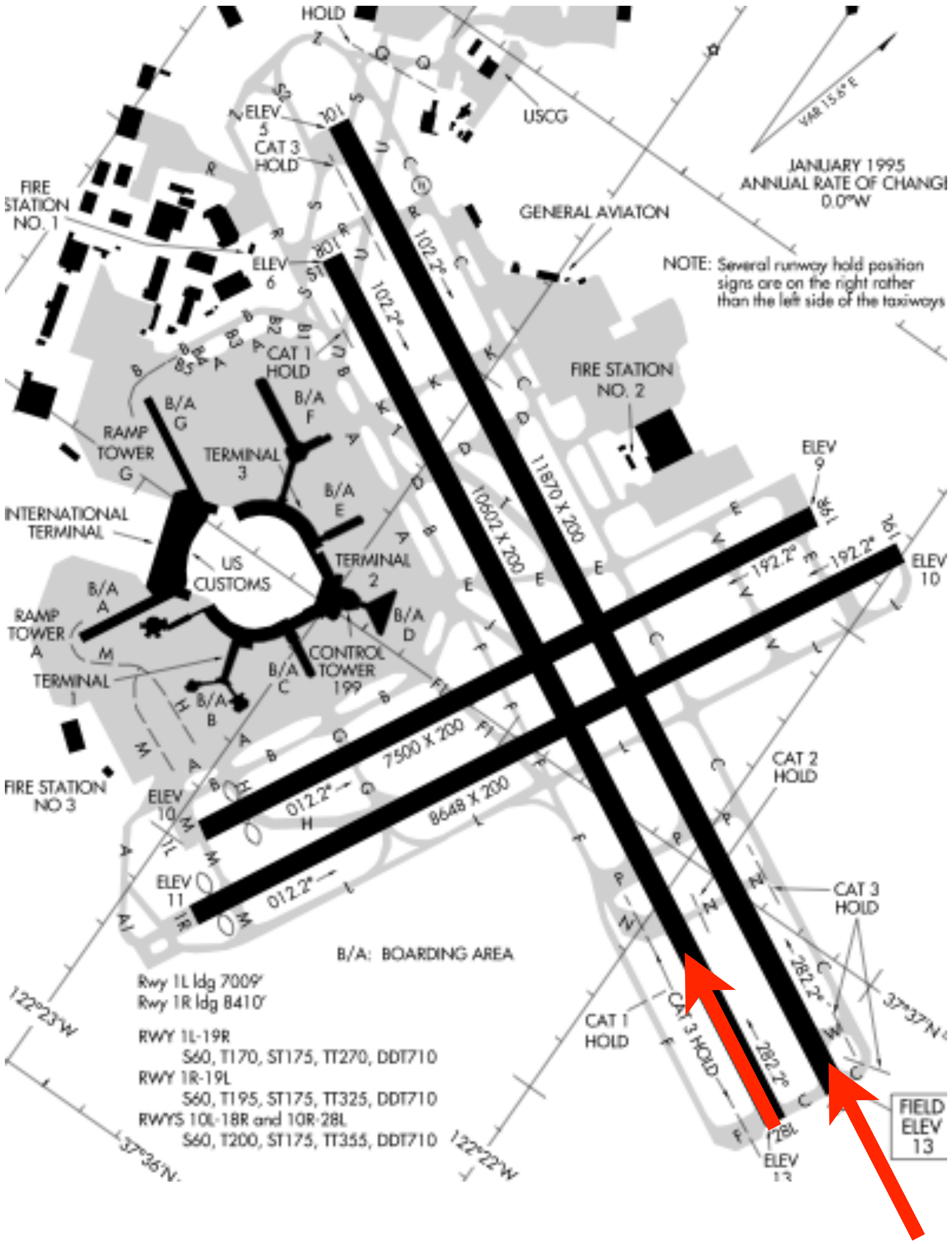

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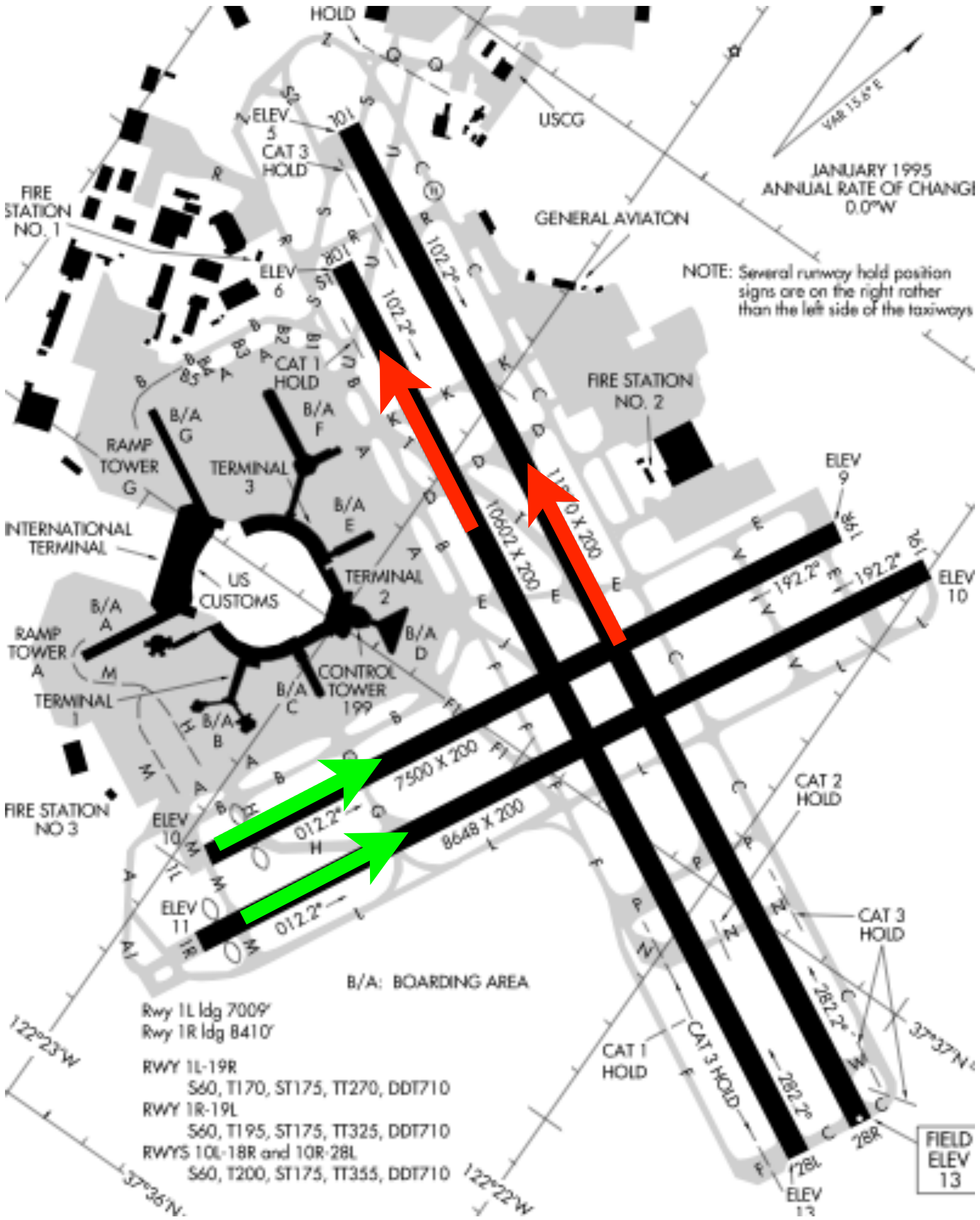
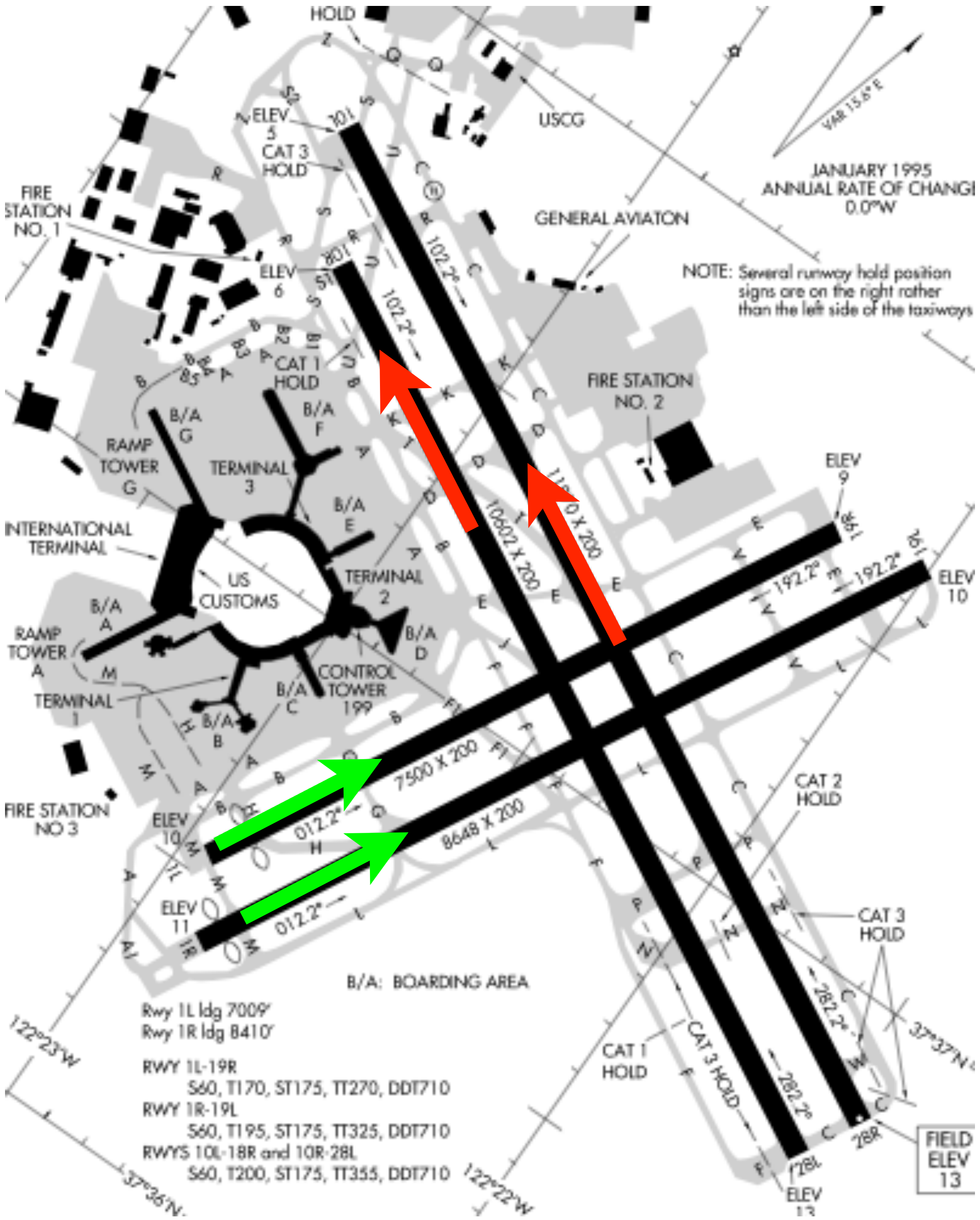
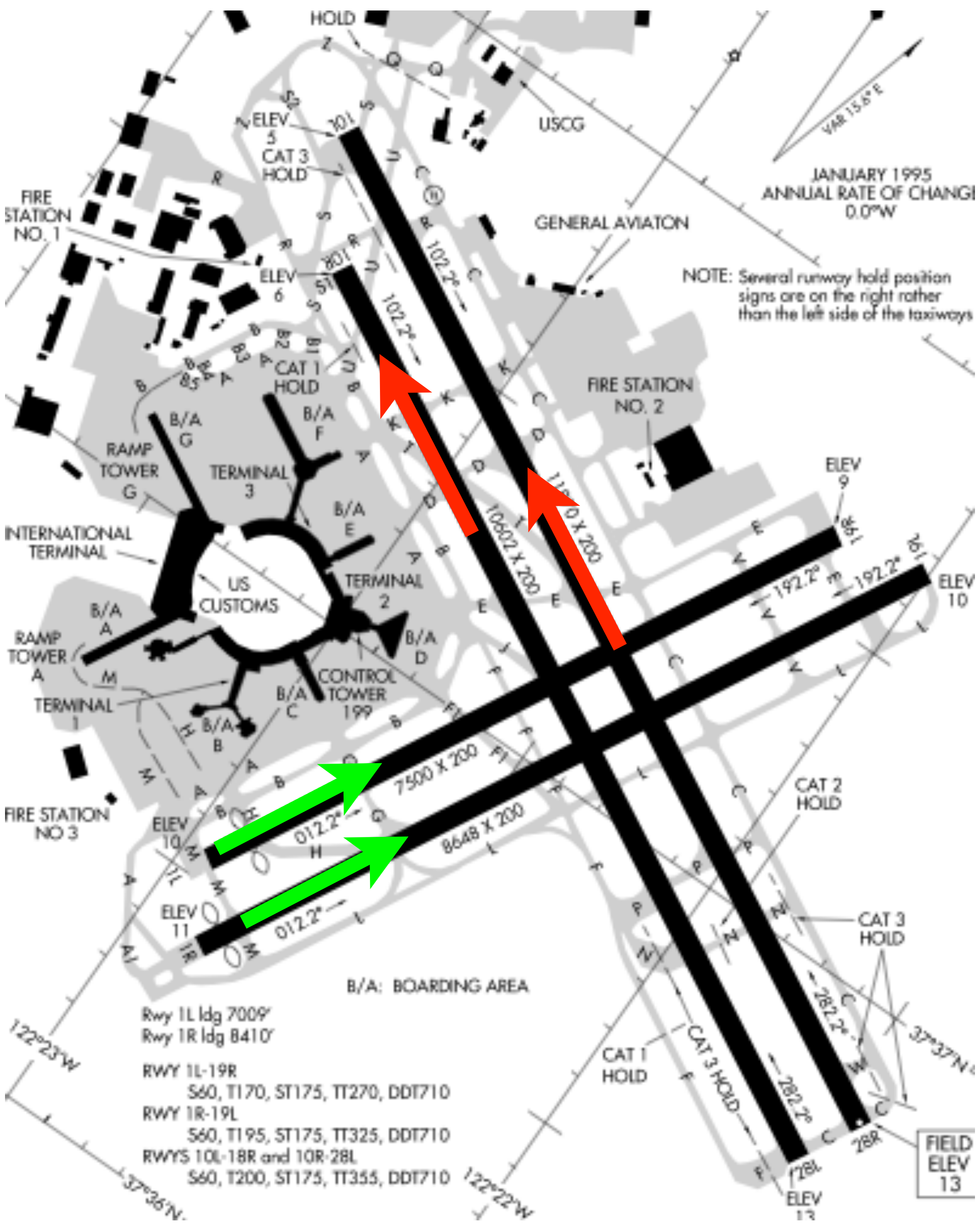


# Paired Arrivals at SFO



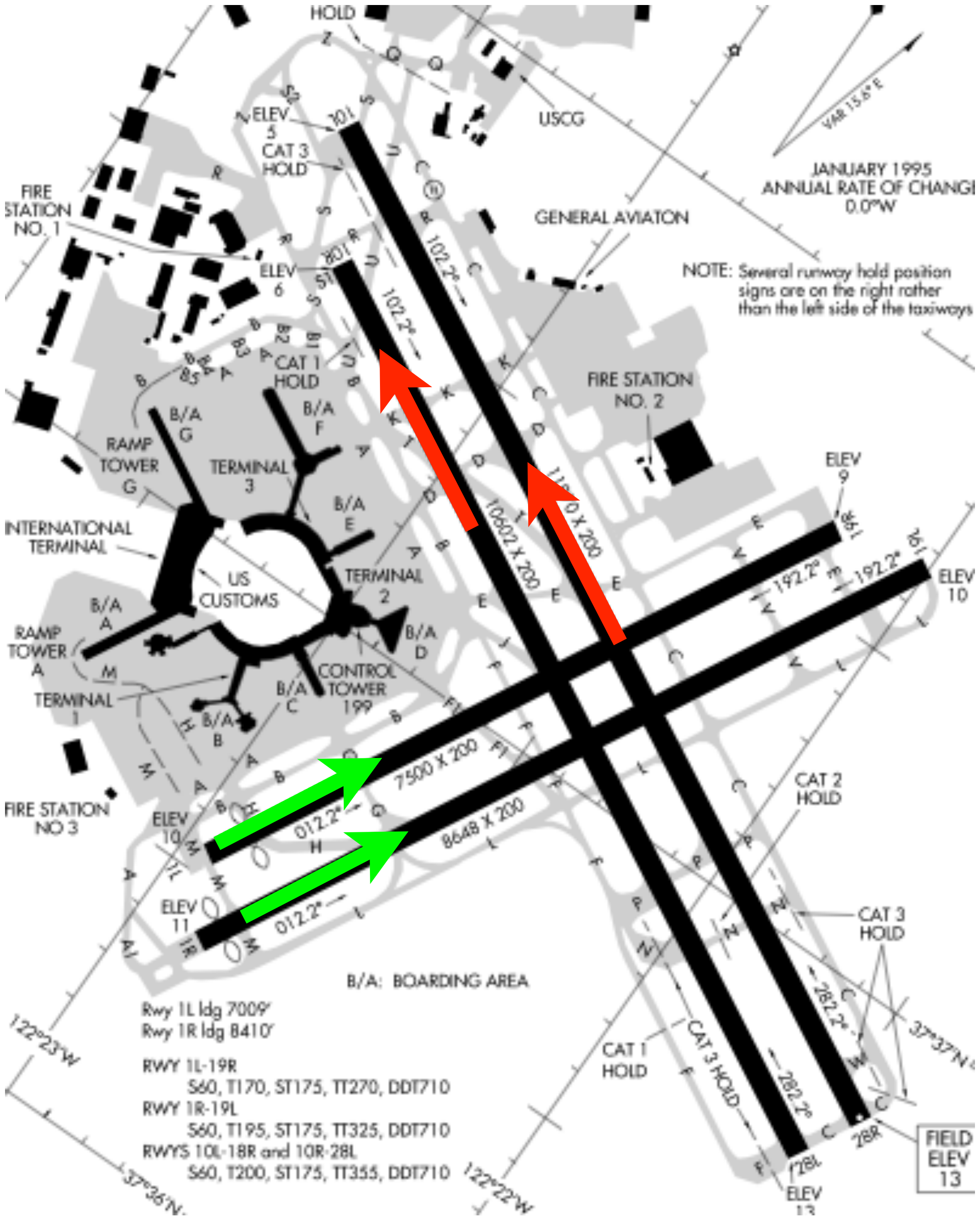

- Situation of heavy traffic for landings and take-offs

# Paired Arrivals at SFO



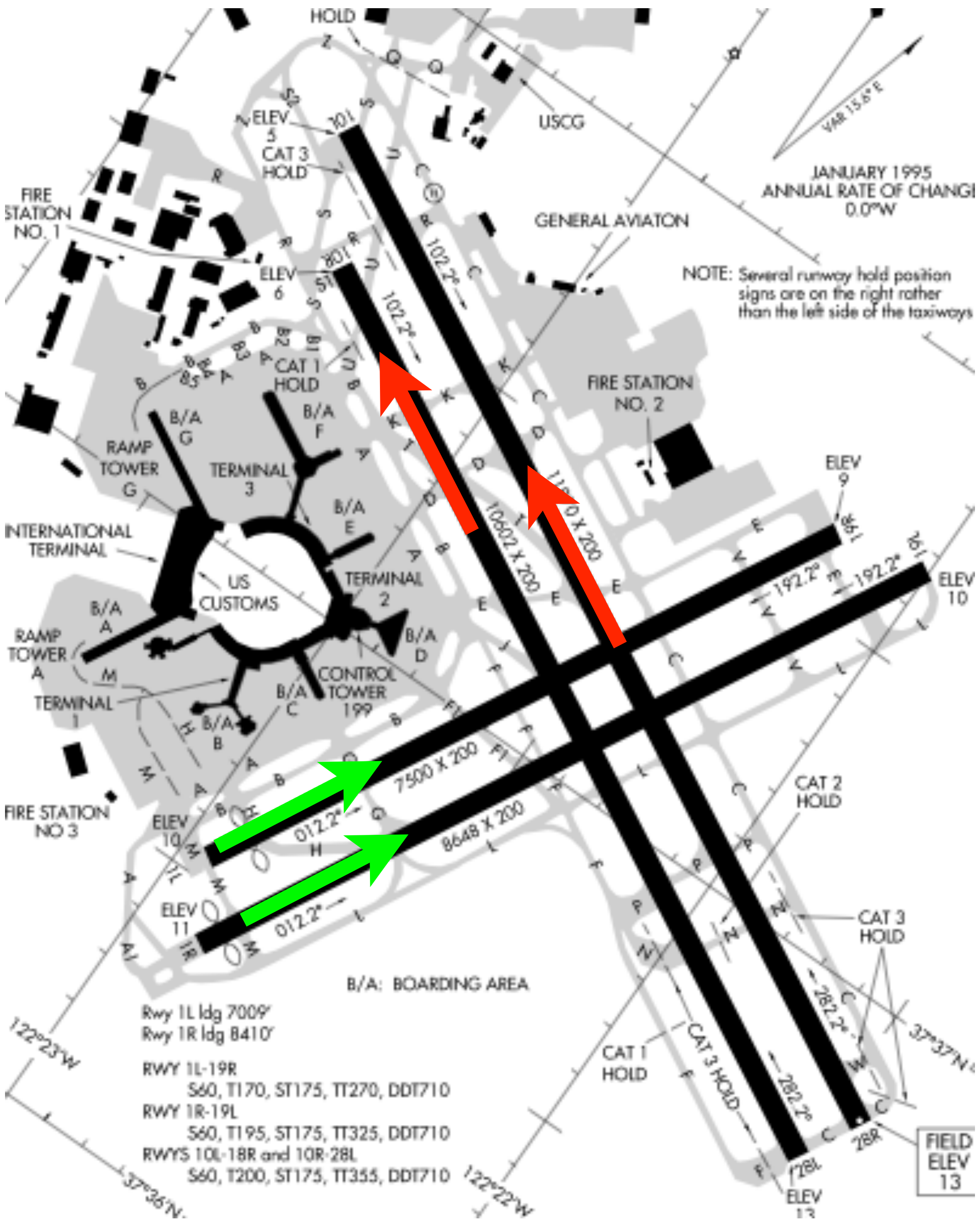

- Situation of heavy traffic for landings and take-offs
- Today: Controllers "guide" aircraft to merging point (5 nmi from 28R)

# Paired Arrivals at SFO




- Situation of heavy traffic for landings and take-offs
- Today: Controllers “guide” aircraft to merging point  (5 nmi from 28R)
- NextGen: Aircraft assigned RTA’s at merging point and descend to the runway

# Paired Arrivals at SFO




- Situation of heavy traffic for landings and take-offs
- Today: Controllers "guide" aircraft to merging point (5 nmi from 28R)
- NextGen: Aircraft assigned RTA's at merging point and descend to the runway
- What is optimal metering headway?

# Approach

# Approach

- Find headway between pairs at the merging point:

# Approach

- Find headway between pairs at the merging point:
  - Enough time between arrival pairs for a departure pair

# Approach

- Find headway between pairs at the merging point:
  - Enough time between arrival pairs for a departure pair
  - Not excessive time separation, resulting in efficiency loss

# Approach

- Find headway between pairs at the merging point:
  - Enough time between arrival pairs for a departure pair
  - Not excessive time separation, resulting in efficiency loss
- Avoid:

# Approach

- Find headway between pairs at the merging point:
  - Enough time between arrival pairs for a departure pair
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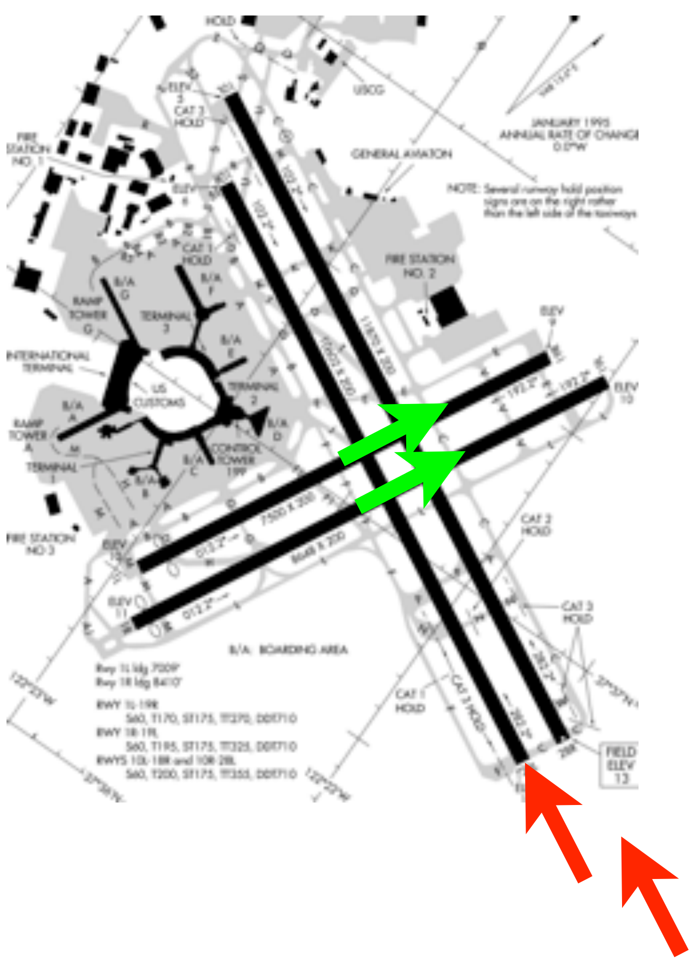
- Avoid:



# Approach

- Find headway between pairs at the merging point:
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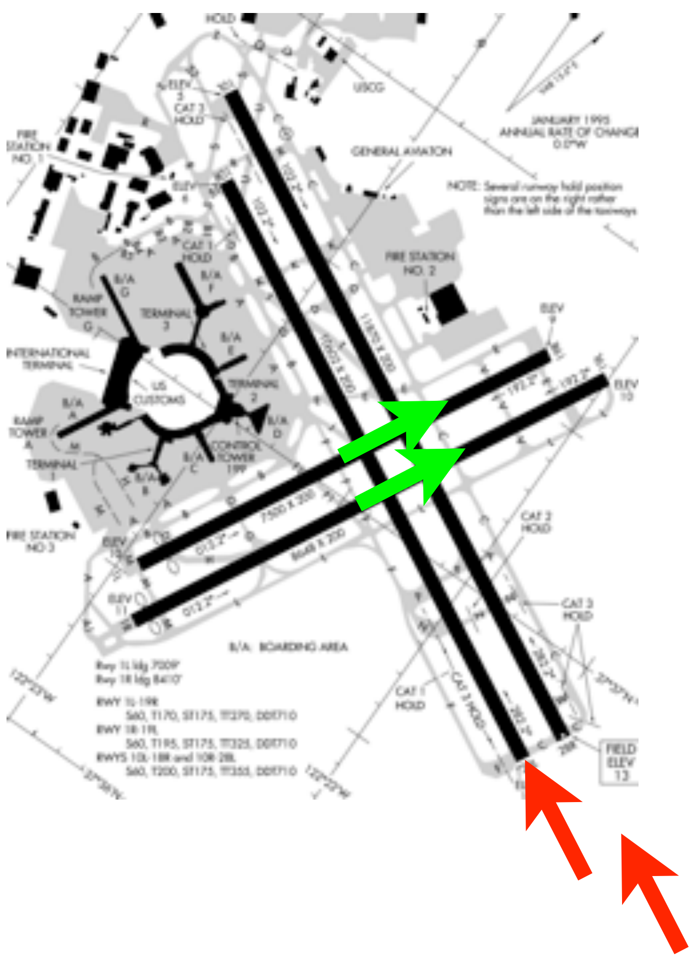
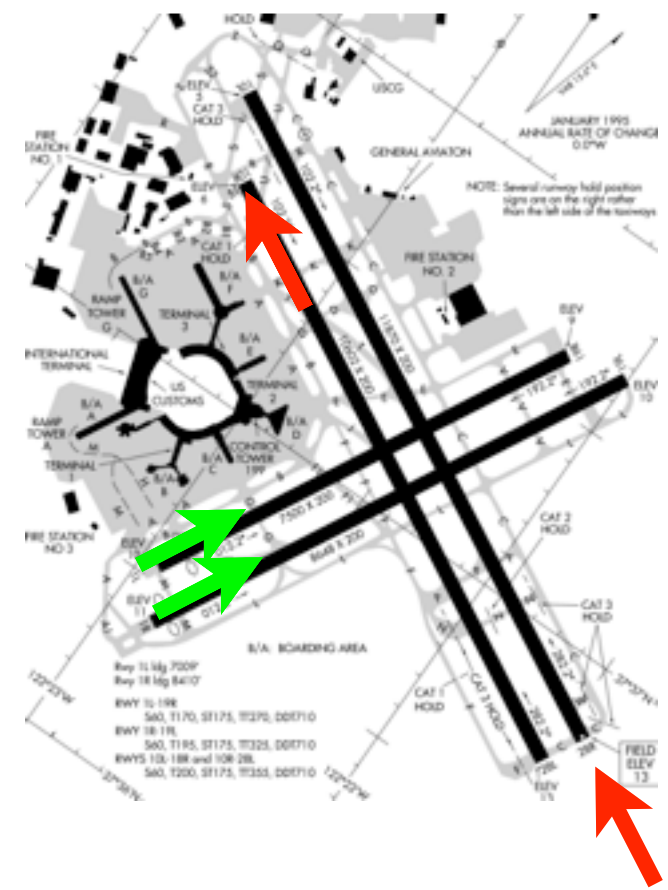
- Avoid:



# Thank you!

# Thank you!

## Questions?