### Trajectory Optimization for Safe, Clean and Quiet Flight

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**Abstract:** Trajectory optimization and 4D navigation are considered as key technologies for the next generation air traffic management. This paper summarized trajectory optimization techniques developed in our laboratory and their applications where we intend to realize safer, cleaner and quitter flight.

Keywords: Trajectory Optimization, Flight Control, Flight Management

### 1. INTRODUCTION

Flight Trajectory Optimization is formulated as an optimal control problem that finds the solution that maximizes or minimizes an objective function within constrained boundaries. Recent development of optimization algorithms and computer capabilities makes it possible to design practical flight trajectories by solving the optimization problems.

Furthermore, concerns about the trajectory optimization have been increasing in the field of the air traffic management. The reasons for this are twofold. Firstly, efficient ATC (Air Traffic Control) has become indispensable due to the increase in aviation traffic. Secondly, the reduction of fuel consumption is called for from the growing concern about an environmental problem. NextGen which is a wide ranging transformation of air transformation system in the U.S.A. states as follows[1],

Trajectory Management: Trajectory management includes any function that affects aircraft trajectory. These functions include trajectory optimization and negotiation with air traffic management, navigation algorithms, delegated aircraft separation applications, or trajectory constraints to avoid weather. The integration of these functions is key to NextGen aircraft functionality.

Another key word in the future ATC is 4D navigation. Many current aircraft have capabilities to generate and follow 3D trajectories with a required time of arrival (3+1/2D trajectory). NextGen states that aircraft should be able to define their required 4D trajectory.

This paper presents trajectory optimization algorithms developed in our laboratory to realize safer, cleaner and quieter flight.

# 2. ONLINE TRAJECTORY OPTIMIZATION FOR EMERGENCY LANDING

Online trajectory optimization methods have been developed to generate 4D flight path for emergency landings. Currently, under airliners' onboard Flight Management Systems (FMS), Flight Management Computers (FMC) calculate a flight plan along an optimal flight path, considering factors such as climate conditions and total weights. However, FMS cannot operate in acute situations such as emergency landings and large shifts of an aircraft's dynamic characteristics associated with failures, because a flight path optimized by FMS consist of routine partial flight paths which are defined by the setting values of airspeed, cruising altitude, angle of direction, and so on.

In general, trajectory optimization problems are formulated as follows:

Minimize: 
$$J = \int_{t}^{t} g(x,u)dt + h(t_{f})$$
Subject to: 
$$dx/dt = f(x,u)$$

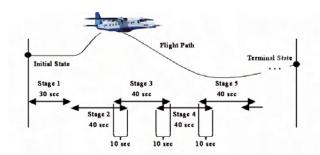
$$x(t_{0}) = x_{0}, x(t_{f}) = x_{f}$$

$$a(x,u) \le 0$$
(1)

where J is an index function to be minimized while satisfying state equations, initial and terminal constraints, and equality/inequality constraints. Although a direct collocation method has been developed to solve the trajectory optimization

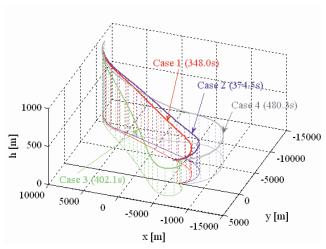
numerically, it cannot be solved in real time since it requires computation time or adequate initial solutions.

To deal with this problem, we developed a multistage approach for the direct collocation method which provides the time histories of control inputs and state variables as a set of nodal points at each time step[2,3]. The multistage approach divides a flight trajectory into multiple segments in which the density of nodal points is adjusted according to the distance from the present position as shown in Fig. 1. The initial flight trajectories are automatically generated in a heuristic manner and they can be used to obtain the initial solutions for the numerical optimization process.



**Figure 1** Multistage division for online flight trajectory optimization

Figure 2 shows the results of numerical simulations where the constraint of the bank angle limitation is changed. Case 1 is the normal case, whereas Cases 2 and 3 are introduced as emergency cases. In Case 2, the right bank is limited to -10[deg]. In Case 3, the right bank is limited to -5[deg]. Figure 2 indicates the three trajectories generated by using the proposed method. Since the right bank is limited, the flight trajectory of Case 2 was computed to avoid a deep right turn. Furthermore, in Case 3, the final turn in the trajectory changed to a left turn. For a comparison, Fig.2 also shows the flight trajectory of Case 4 in which the right bank angle is limited to -5[deg], which is the same as that of Case 3. In addition, the final turn of the preliminary flight path is limited to a right turn in Case 4.



**Figure 2** Optimal Flight Trajectories for Different Bank Angle Constrains.

In order to track the optimized trajectory generated, an autopilot with an auto throttle system must be designed for a 4D navigation system. We developed a nonlinear dynamic inversion controller with a singular perturbation method (NDI-SPM). NDI can realize a simple closed system by using inverted model equations, which can be performed without complicated gain scheduling. However, it is not practical to obtain the inverted model from the complete set of dynamic equations. If the system can be partitioned into several timescale systems, it can also be divided according to its time scale. In this study, we assume that the aircraft angular velocity increases to a greater rate than the attitude and velocity. Thus, the SPM can be applied to simplify the NDI controller and increase the controller robustness. Fig. 3 illustrates the block diagram of the NDI-SPM flight controller.

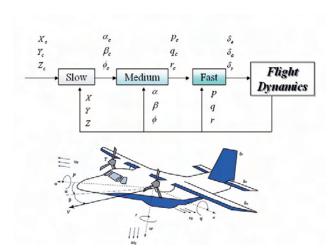
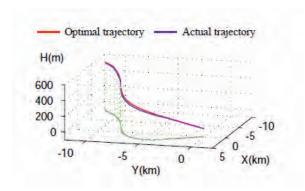


Figure 3 NDI-SPM Controller for 4D Navigation

We conducted flight testing using an experimental aircraft MuPAL- $\alpha$  owned by JAXA (Japan Aerospace eXploration Agency). In the flight testing, an

optimized flight trajectory generated by Online Optimization was tracked automatically by NDI-SPM. Figure 4 demonstrates the agreement between the required trajectory and the achieved trajectory in the flight test.



**Figure 4** Flight Test Results of Automatic Tracking of Optimal Flight Trajectory

# 3. ONLINE TRAJECTORY OPTIMIZATION FOR LOW NOISE LANDING

The online trajectory optimization method was applied to obtain helicopter flight trajectories to reduce ground noise in the landing approach. It is well-known that a major problem in helicopter operations is the noise level. The large noise that spreads throughout the environment surrounding a flight path is arguably one of the main reasons that helicopters are not used widely.

The objective function is formulated by using the estimated ground noise level. Flight testing including the noise measurement was carried out to evaluate the proposed method[4]. Figure 5 shows MuPAL-  $\epsilon$  owned by JAXA used in this flight test. Figure 6 illustrates the comparison of flight trajectories. Case1 and Case2 are optimized flight trajectories and are compared with an ordinary flight.



Figure 5 Experimental Helicopter MuPAL- ε (JAXA)

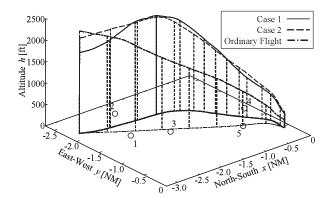


Figure 6 Comparison of Flight Paths

Case 1 is the optimized flight fright trajectory with the upper limit of climb rate set at 800 fpm. Since the Case 1 indicates the ascent flight before the descent which should be avoided because pilots are not accustomed to such a maneuver. Case 2 modified the upper limit of the climb rate. It should be noted that optimal trajectory is the best from a mathematical perspective, but might be not suitable for the pilot's control. Therefore, several additional constraints should be introduced.

## 4. MULTI AIRCRAFT DESCENT MANAGEMENT

Fuel saving in civil aviation is considered indispensable to both cost reduction and CO<sub>2</sub> emission abatement. We analyzed multiple civil aircraft descent trajectories by the use of optimization[5]. In particular, the arrival scheduling of two aircraft was investigated. Heavy and medium weight aircraft were to descent from cruising altitude of 33000 feet near the top of descent to a common altitude of 5000 feet. The two aircraft arrive at the end point with a certain predefined time separation. Depending on the flight time and the order of arrival of the aircraft, significant differences in total fuel consumption were obtained.

In numerical optimization, the time axis is divided into N elements. Control variables are the lift and thrust coefficients,  $C_L$  and  $C_T$  which are assumed constant in each element. By estimating the state variables, such as the position, the velocity, the mass, and the flight path angle at each nodal point along the time axis, an optimization problem can be formulated. The objective function is fuel consumption and the upper limit of the descent path angle is 3 degree. Figure 7 illustrates the optimized single aircraft trajectory which confirms the fact that flying higher for longer results in lower fuel consumption.

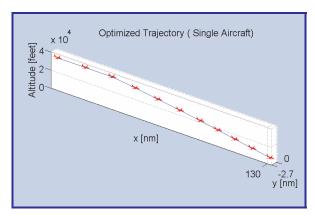


Figure 7 Optimized Single Aircraft Trajectory

Nest, the descent management of medium and heavy aircraft, B737 type and B747 type is considered. The total fuel consumption of the two aircraft is minimized with the consideration of the minimum separation of the two aircraft and the time separation at the final waypoint.

The computational results are summarized in Figure 8. It shows the total fuel burnt versus the flight time and the order of arrival. As seen from the figure, when the flight time is less than 2000 s (the yellow zone of the figure), less fuel is consumed when the B737 arrives at the last waypoint prior than the B747. However, when the flight time is more than 2000 s (the green zone of the figure), B747 should be the first to arrive. According to these results, the heavy aircraft is not necessarily the one to fly shorter. In fact, an optimal solution is acquired when in the two aircraft case the flight time of the B747 is closer to its own optimal flight time when the B737 is not considered. Note that this computation was carried out in an off-line manner and should be realized in real time computation in the next step.

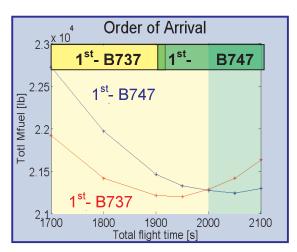


Figure 8 Order of Arrival of Two Aircraft

### 5. CONCLUSION

The realization of optimization is strongly required for the future air traffic management. This paper briefly summarized the numerical optimization techniques developed in our laboratory. The following three applications are presented for demonstration. 1) The online trajectory optimization and automatic 4D trajectory tracking is developed for emergency landing control. Flight tests are carried out to confirm the availability of the method. 2) The trajectory design of helicopter for low noise landing is investigated. Flight tests show the usefulness of the optimized flight. 3) Multiple aircraft descent management is analyzed to indicate the order of two aircraft decent. The authors believe that the trajectory optimization should be investigated to realize safer, cleaner and quieter flight in the next generation of flight management control system.

### 6. ACKNOWLEDGMENTS

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