Frequency of Rare Event Occurrence

ICAO collision risk model for separation minima

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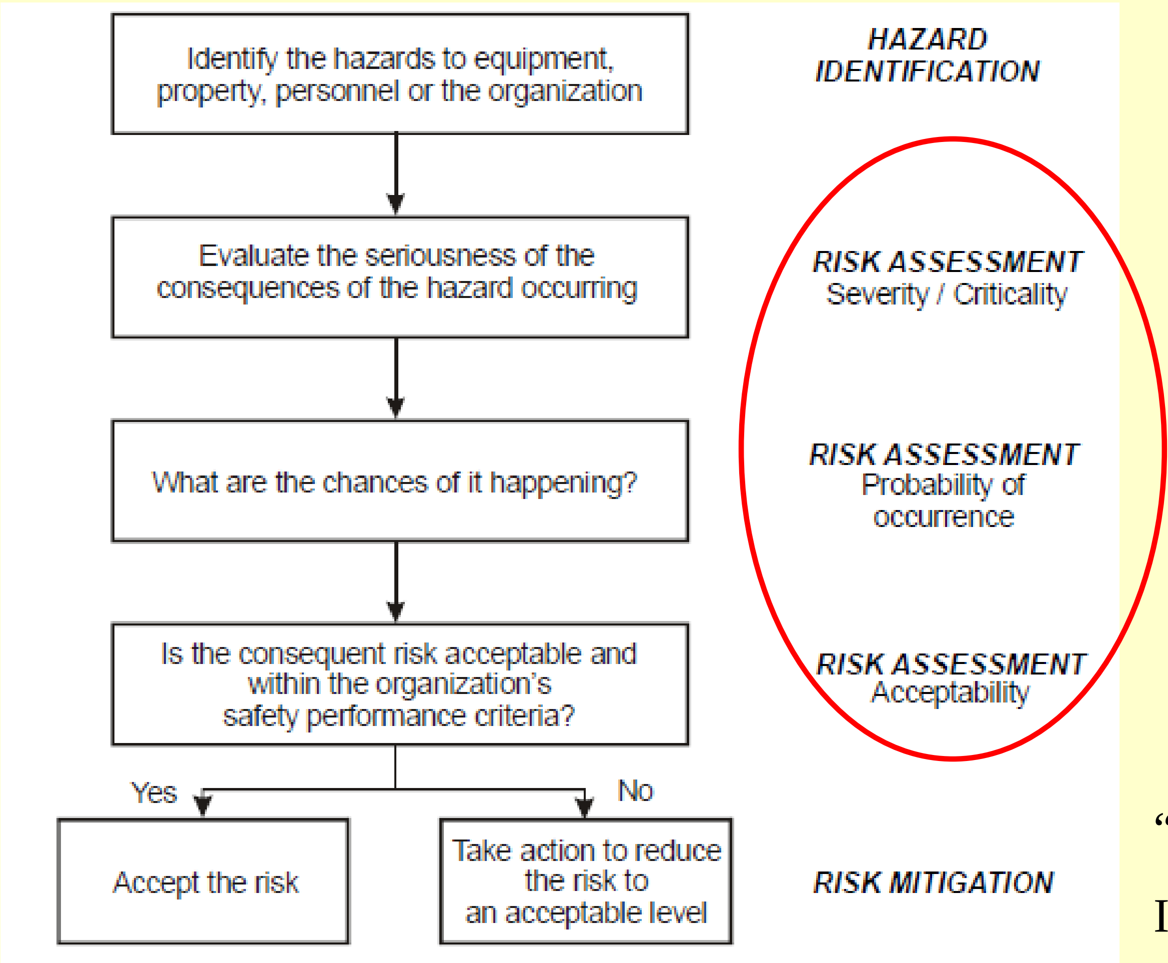
Safety (Definition)

Safety is the state in which the risk of harm to person or property of damage is reduced to, and maintained at or below, an acceptable level through a continuing process of hazard identification and risk management

"Safety Management Manual", ICAO Doc 9859-AN/953

Safety is considered to be a state as well as action.

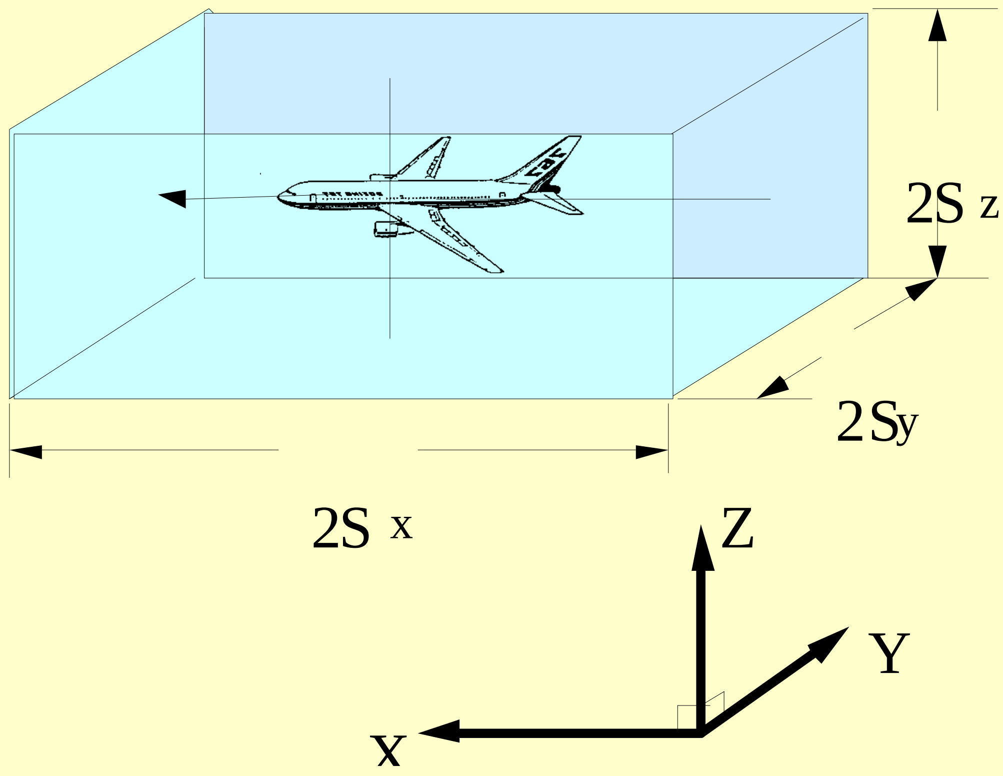
Safety Management Process



"Safety Management Manual",

ICAO Doc 9859-AN/953

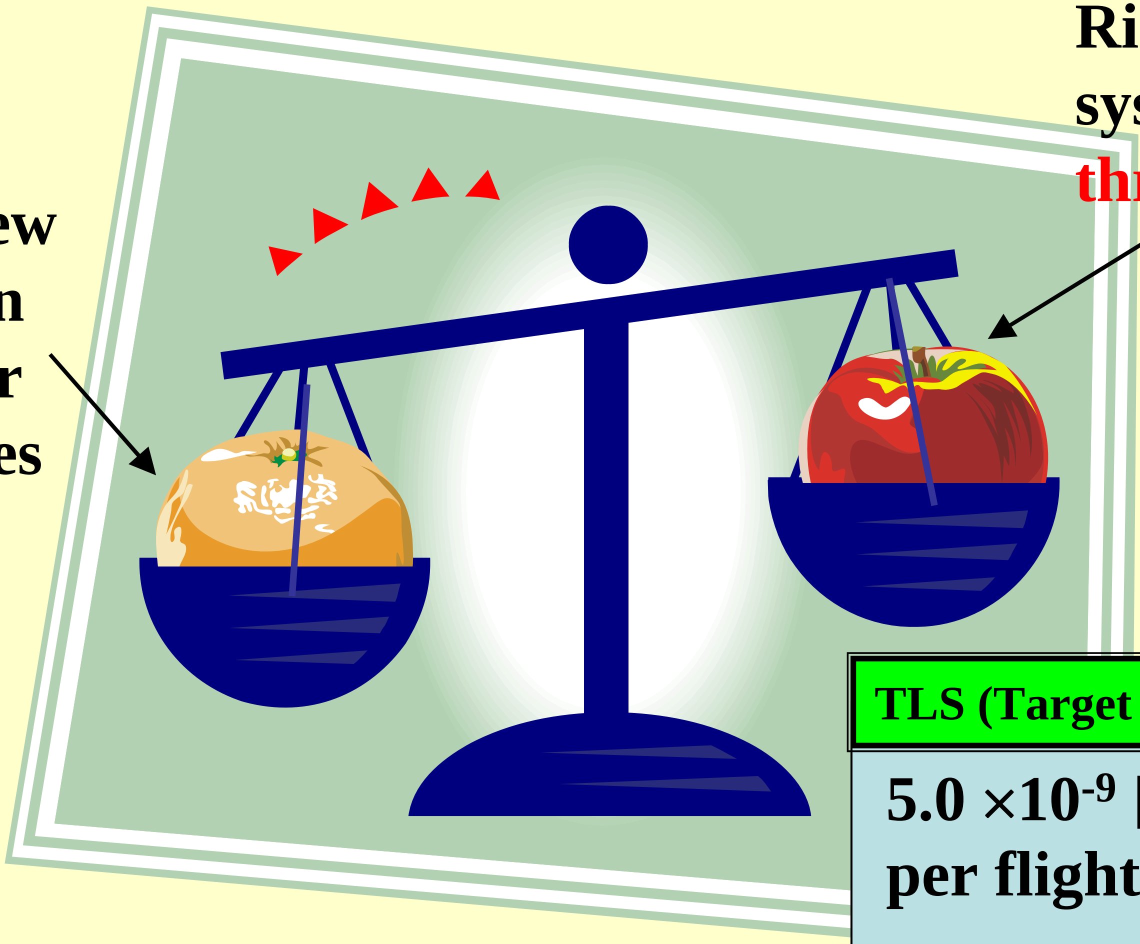
Separation minima



Separation minimum of aircraft pairs for tactical collision avoidance.

Risk Comparison

Risk of new separation minima or procedures



Risk of reference system or absolute threshold

TLS (Target Level of Safety)

5.0×10^{-9} [fatal accidents per flight hour]

ICAO collision risk model

ICAO Collision Risk Model (CRM)
Mathematical models used in predicting risk of mid-air collision

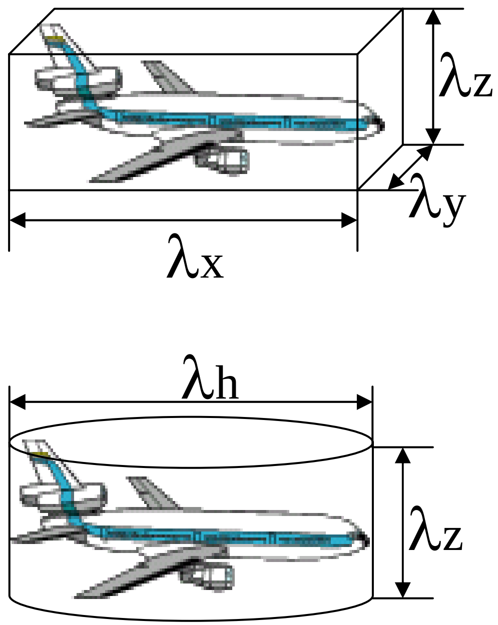
- **Reich CRM** — Time-invariant error model

  Reich, J., “Analysis of long-range air traffic system: separation standard”, Journal of the Institute of Navigation vol. 19 (1996)

- **Rice CRM** — Time-invariant & time-variant error model

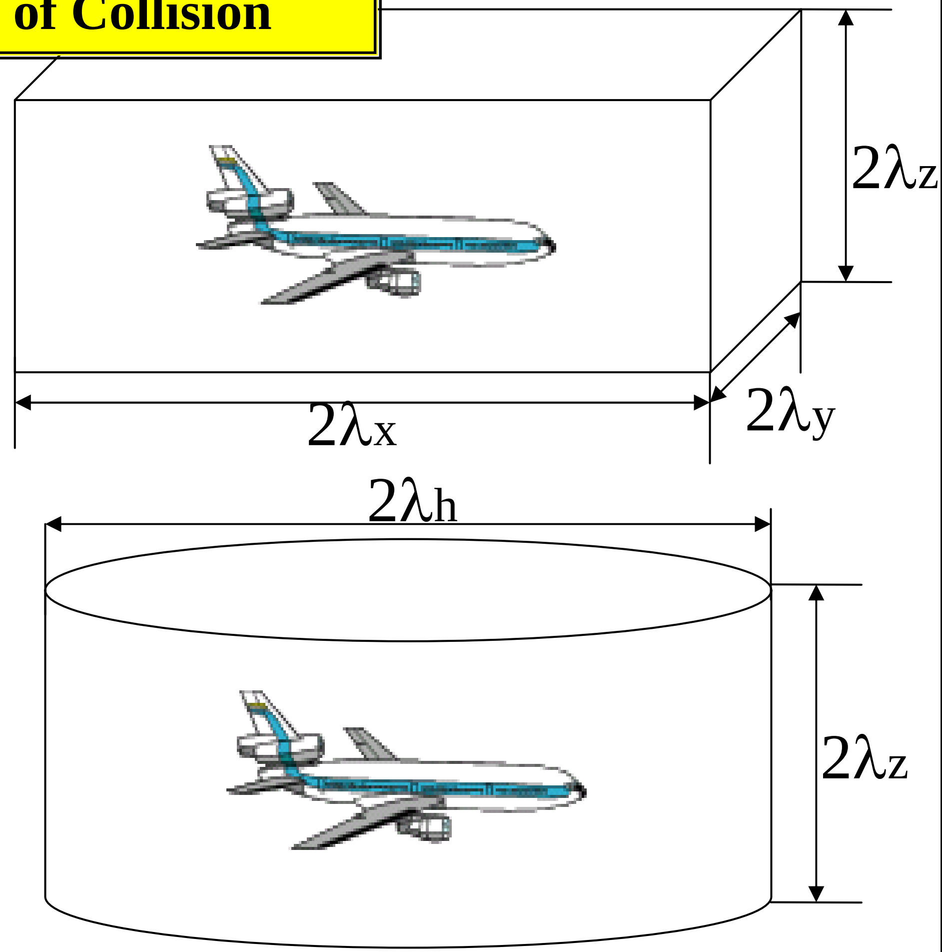
  “A unified framework for collision risk modelling in support of the manual on airspace planning methodology for further application”, ICAO Circ 319-AN/181

Area of Collision



Simplification of aircraft shape

Area of Collision



Rice formula

$$\Pr\{\text{collision during } [t_0, t_1]\} = \int_{t_0}^{t_1} \Psi(u)\,du$$

$$\Psi(t) = \iint_{\partial\Omega} \left(\iiint f_t(X,V)(\vec{n} \cdot \vec{V})^+ \, dV \right) dS$$

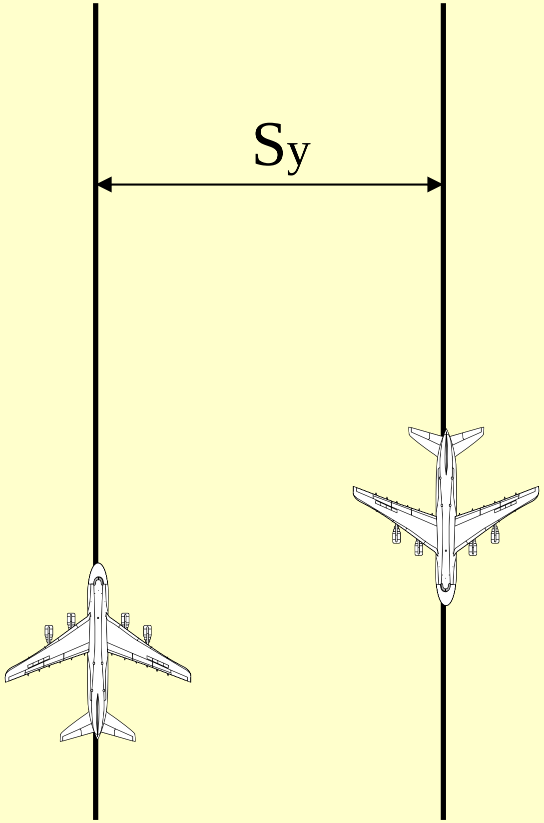
Ω : area of collision

$\partial\Omega$: the boundary of Ω

$\vec{n}$: the normal vector of the boundary surface

$f_t(X,V)$: the probability density function on the phase space (X,V) at the given time instance t

Parallel Route Case

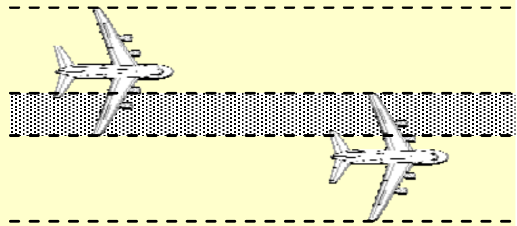


$$N_{ay} = N_x P_y(S_y) P_z(0)\left(1 + \frac{\lambda_x}{\lambda_y}\frac{\overline{|\dot{y}|}}{\overline{|\dot{x}|}} + \frac{\lambda_x}{\lambda_z}\frac{\overline{|\dot{z}|}}{\overline{|\dot{x}|}}\right)$$

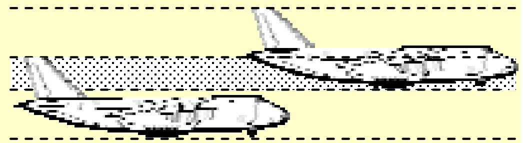
Description

Notation	Description
N_{ay}	Collision risk defined as the expected number of fatal accidents due to loss of lateral separation per flight hour
N_x	Passing frequency. twice of the number of passing events divided by total flight hour
$P_y(S_y)$	Lateral overlap probability. Probability that a pair of aircraft being nominally and laterally separated by S_y NM overlaps laterally.
$P_z(0)$	Vertical overlap probability. Probability that a pair of aircraft flying at the same flight level overlaps vertically.
λ_x, λ_y, λ_z	Average aircraft length, average wing span and average aircraft height
$\overline{\lvert\dot{x}\rvert}$ $\overline{\lvert\dot{y}\rvert}$ $\overline{\lvert\dot{z}\rvert}$	Average longitudinal, lateral and vertical relative velocity of a pair of aircraft which are about to collide.

Lateral overlap



Vertical overlap



Special Case
(flow rate and cruise speed are given)

$$N_{ay} = \left(\sum_{i=1}^{N-1} \sum_{j=1}^{K} \frac{2 n_{i,j} n_{i+1,j} |V_{i+1,j} - V_{i,j}|}{|V_{i+1,j}||V_{i,j}|} P_y(S_{i,i+1}) P_z(0) \right.$$
$$\left. \times \left(1 + \frac{v_y/(2\lambda_y)}{|V_{i+1,j} - V_{i,j}|/(2\lambda_x)} + \frac{v_z/(2\lambda_z)}{|V_{i+1,j} - V_{i,j}|/(2\lambda_x)} \right) \right)$$
$$\Bigg/ \left(\sum_{i=1}^{N-1} \sum_{j=1}^{K} \frac{n_{i,j} |V_{i+1,j}| + n_{i+1,j} |V_{i,j}|}{|V_{i+1,j}||V_{i,j}|} \right)$$

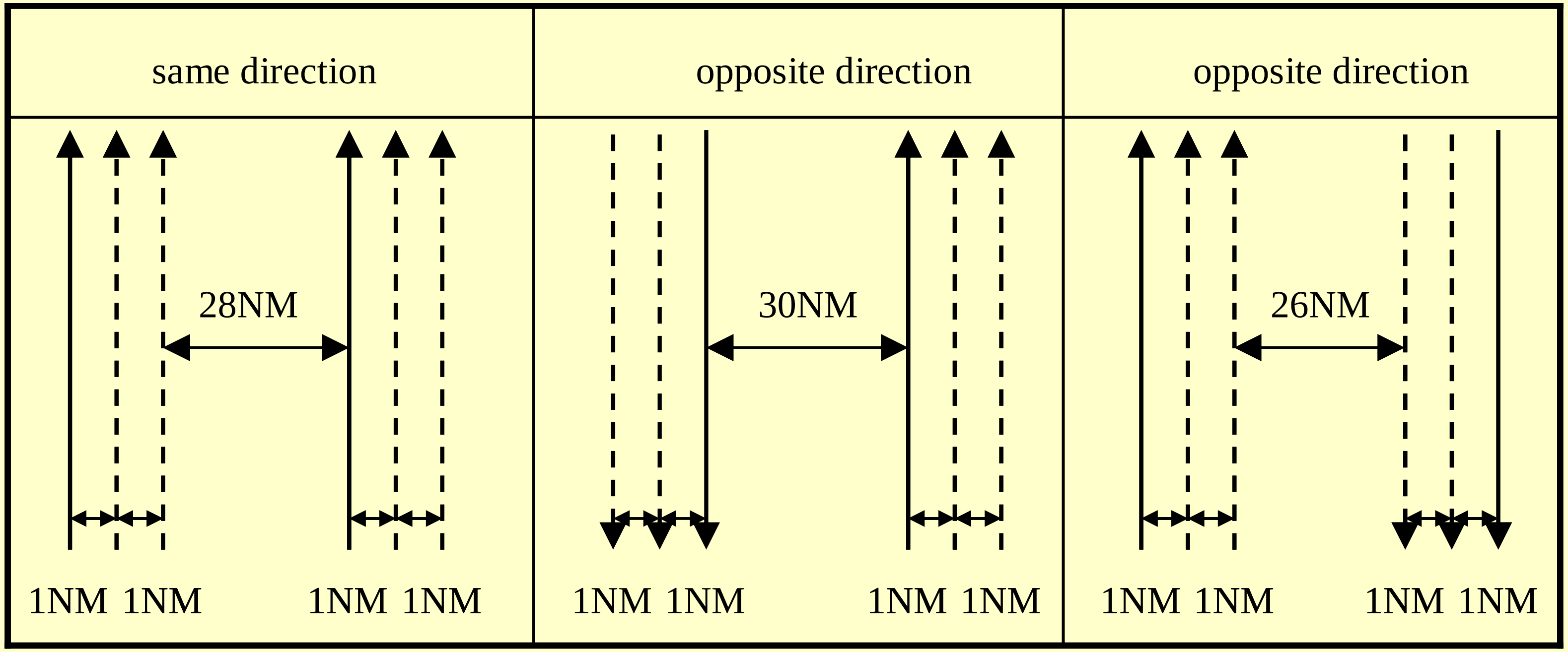
Remark:
Risk is approximately twice if the flow rates are doubled

$n_{i,j}$: flow rate of aircraft on route R_i at flight level H_j
$V_{i,j}$: speed of aircraft on route R_i at flight level H_j

Effect of SLOP application

SLOP

lateral offset 1NM or 2NM to the right

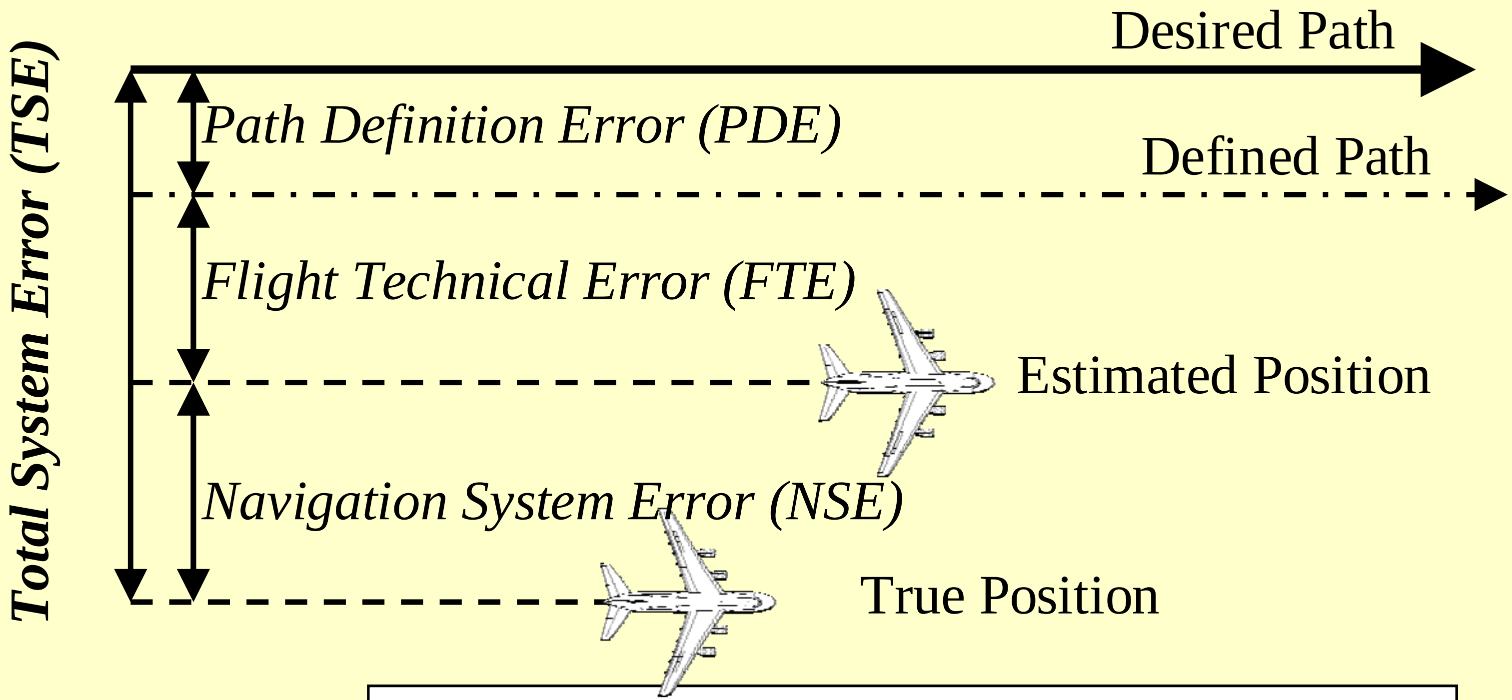


Performance-based Navigation

The Performance Based Navigation (PBN) concept specifies RNAV system performance requirements in terms of accuracy, integrity, availability, continuity and functionality needed for the proposed operation in the context of a particular airspace concept, when supported by the appropriate navigation infrastructure.

"Performance-Based Navigation Manual", ICAO Doc 9613-AN/937

Total System Error



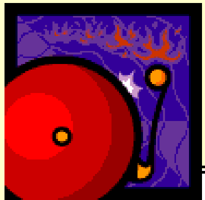
$f_{TSE}(u)$: probability density function of total system errors (TSEs)

“Performance-Based Navigation Manual”, ICAO Doc 9613-AN/937

RNAV-X vs RNP-X

The expression ‘X’ refers to the lateral navigation accuracy of X nautical miles (NM) that is expected to be achieved at least 95 percent of the flight time by the population of aircraft operating within the airspace, route and procedure

$$\int_{-X}^{X} f_{TSE}(x)dx = 0.95$$



RNP-X aircraft has a monitoring and alerting function that ensures that the probability that the total system error of each aircraft exceeds 2 times X without annunciation is less than 10^{-5}.

$$\int_{-2X}^{2X} f_{TSE}(x)dx = 1 - 10^{-5}$$

SASP MSG TSE model

RNAV-X aircraft

$$f_{TSE}(y) = \frac{\exp(-|y|/\lambda)}{2\lambda}$$

Double Exponential (DE) distribution

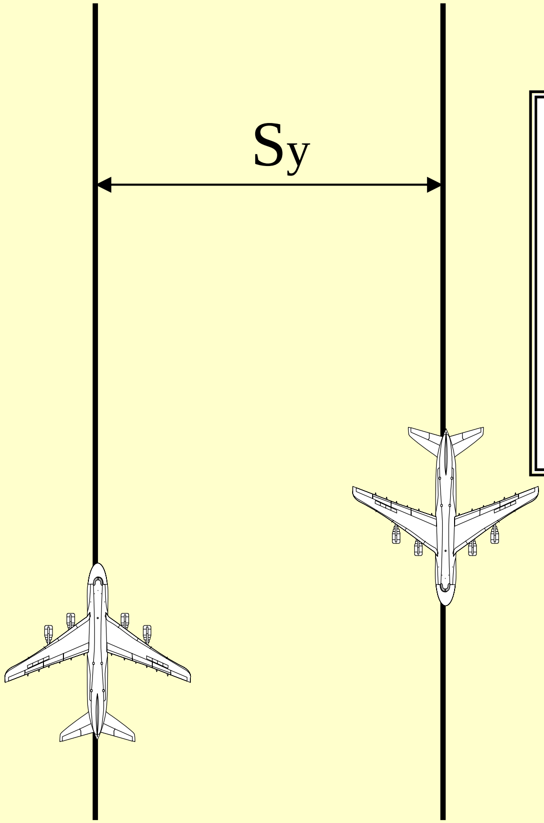
RNP-X aircraft

$$f_{TSE}(y) = \frac{\exp(-y^2/(2\sigma^2))}{\sqrt{2\pi}\sigma}$$

Gaussian distribution

"Report of the Mathematicians Sub Group meeting at the SASP WG/WHL/10 meeting", ICAO SASP/WG/WHL/10, Gold Coast, November 2006

Lateral Overlap Probability



$$P_y(S_y) = \int_{S_y-\lambda_y}^{S_y+\lambda_y} \int_{-\infty}^{+\infty} f_{TSE}(y+u) f_{TSE}(u)\,du\,dy$$

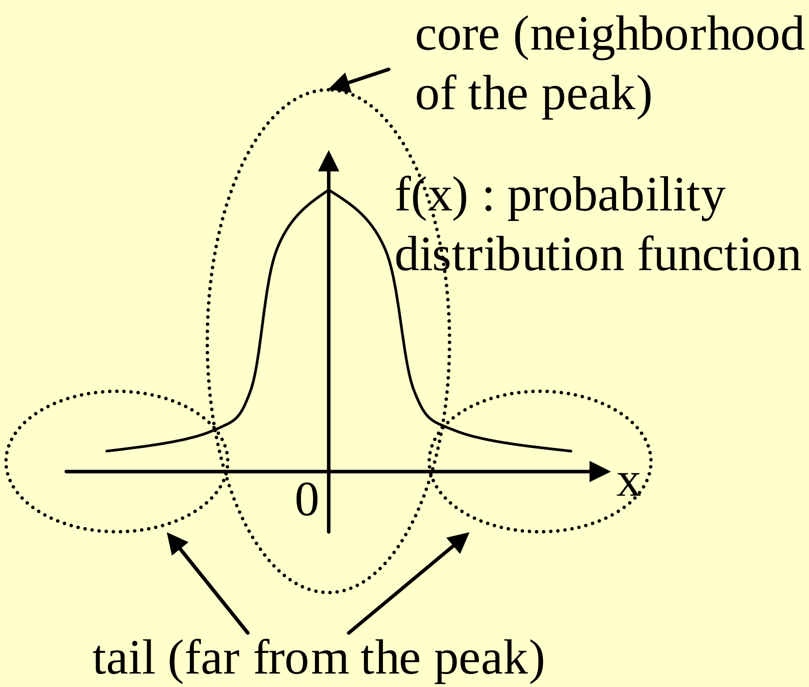
$$\cong 2\lambda_y \int_{-\infty}^{+\infty} f_{TSE}(S_y+u) f_{TSE}(u)\,du$$

$f_{TSE}(u)$: probability density function of total system errors (TSEs)

Convolution

		$\int_{-\infty}^{+\infty} f_{TSE}(S_y + u) f_{TSE}(u) du$ for X=4			
Satisfied Condition		$\int_{-X}^{X} f_{TSE}(x)dx = 0.95$		$\int_{-2X}^{2X} f_{TSE}(x)dx = 1-10^{-5}$	
Distribution Type		DE	Gaussian	DE	Gaussian
Sy (NM)	26	1.31×10^{-8}	3.30×10^{-19}	6.86×10^{-17}	2.43×10^{-24}
	27	6.40×10^{-9}	1.37×10^{-20}	1.53×10^{-18}	3.95×10^{-26}
	28	3.13×10^{-9}	5.05×10^{-22}	3.41×10^{-18}	5.45×10^{-28}
	29	1.53×10^{-9}	1.65×10^{-23}	7.58×10^{-19}	6.56×10^{-30}
	30	7.45×10^{-10}	4.77×10^{-25}	1.68×10^{-19}	6.70×10^{-32}

Tail of Distribution



Collision risk is dominated mainly by the tails of distributions

Extreme Value Theory

Extreme value theory claims that the conditional probability $\Pr\{Y<y|Y>u\}$ of distributions satisfying some technical assumptions approximately follows a generalized Pareto distribution (GPD) when u is large enough. The cumulative distribution function of a generalized Pareto distribution is given by

$$H(y) = 1 - \left(1 + \xi \frac{y}{\sigma}\right)^{-1/\xi}, \quad 1 + \xi y / \sigma > 0$$

When the shape parameter $\xi<0$, the GPDs are Beta distributions. They are exponential distributions and Pareto distributions in the case where $\xi=0$ and $\xi>0$, respectively.

DE vs Gaussian

- Convolution value (and lateral overlap probability) is more than 10^7 smaller in Gaussian case than DE case.
- We do NOT have any evidence that TSE distribution does NOT follow Gaussian distribution, but we do NOT have any evidence that it DOES follow Gaussian distribution.
- GPD is an exponential distribution in case where the original distribution is Gaussian.

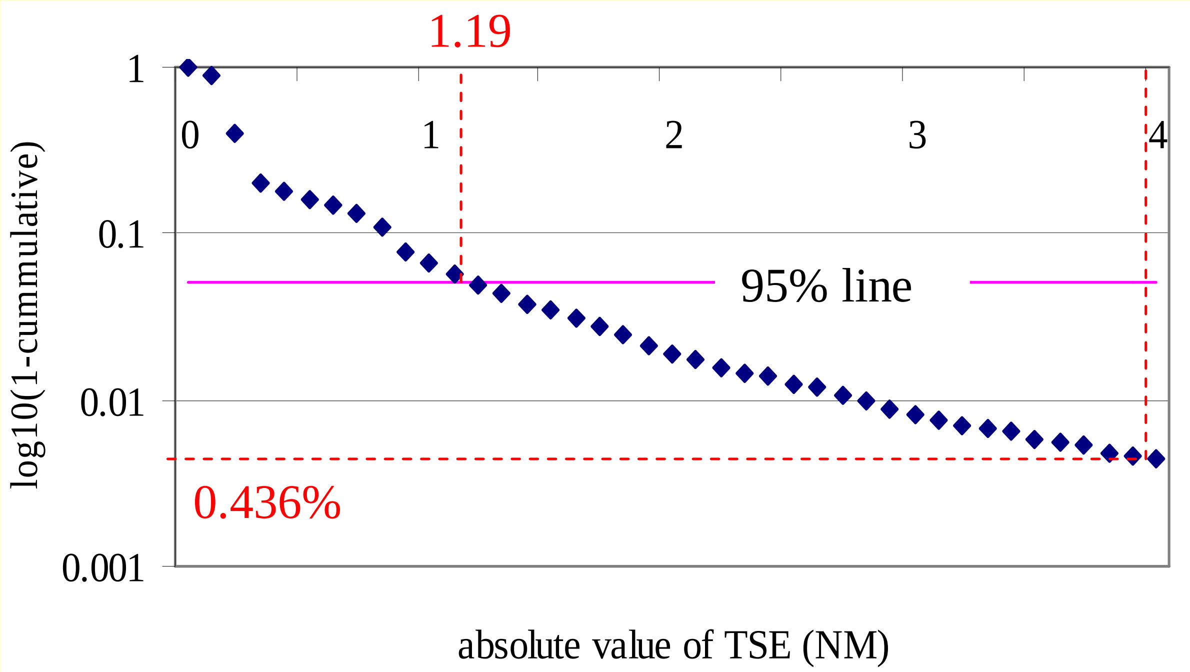
Conclusion

DE is better for the model of TSE.

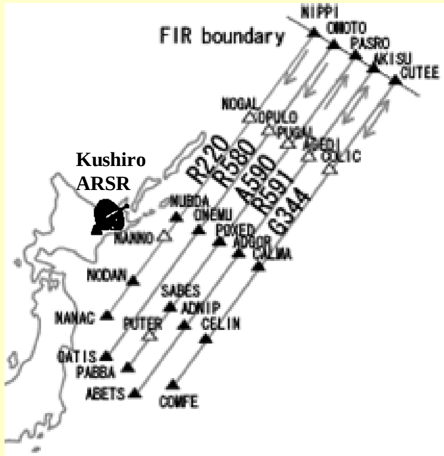
What we should do in case where sufficient data set is not available

- to make an assumption on the type of distribution and conduct a safety assessment based on this assumption. This assumption should seem to be sufficiently conservative
- to continue monitoring to check whether a new hazard is found and whether the assumption made is truly conservative after the implementation of a new separation minimum or a procedure,

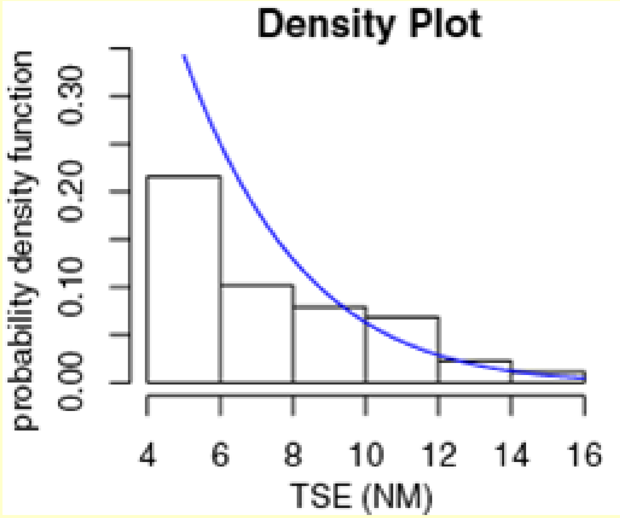
TSE distribution (observed)

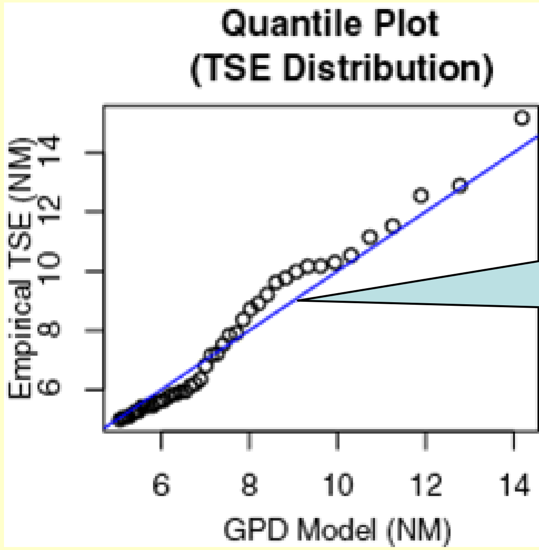


No. of data : 23,639
Max. 12.9 (NM)
Min. -15.9 (NM)
AVG. -0.183 (NM)
STD. 0.651 (NM)
SKEW -3.17
KURT 69.6



Peak over Threshold





Small size of given data set

$$H(y) = 1 - \left(1 + \xi \frac{y}{\sigma}\right)^{-1/\xi} \quad \begin{cases} \xi = -0.103 \\ \sigma = 2.92 \end{cases}$$

Summary

- ICAO developed mathematical models which are utilized for the quantitative safety assessment in the determination of separation minima. They are called collision risk models (CRMs).
- Reich formula is applicable only for time-invariant case but Rice formula is an extension version of Reich formula and it is applicable for both time-invariant and time-variant case.
- We developed a collision risk formula for a parallel route system in case where the flow rate and the aircraft speed on each route at each flight level is constant.

Summary (cont'd)

- Double exponential (DE) distribution is compared with Gaussian distribution. We should choose a conservative model not to underestimate the risk. The author claimed that DE distribution is better for the model of TSE distribution.
- We should continue monitoring to check whether a new hazard is found and whether the assumption on the distribution model is truly conservative for the risk estimation.
- Generalized Pareto distribution (GPD) is also introduced and applied to TSE data set. GPD is a good tool to investigate the shape of tails if the sufficient size of data is obtained.

Thank you for your attention!!