Kyushu University 2011
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Flight Trajectory Optimization Tool with Dynamic Programming Developed for Future Air Transportation System

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Outline

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2. Trajectory Optimization by Dynamic Programming
   - Definition of optimal control problem
   - Advantages and disadvantages of DP
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   - Computational time reduction technique to overcome "Curse of Dimensionality".
4. Conclusions

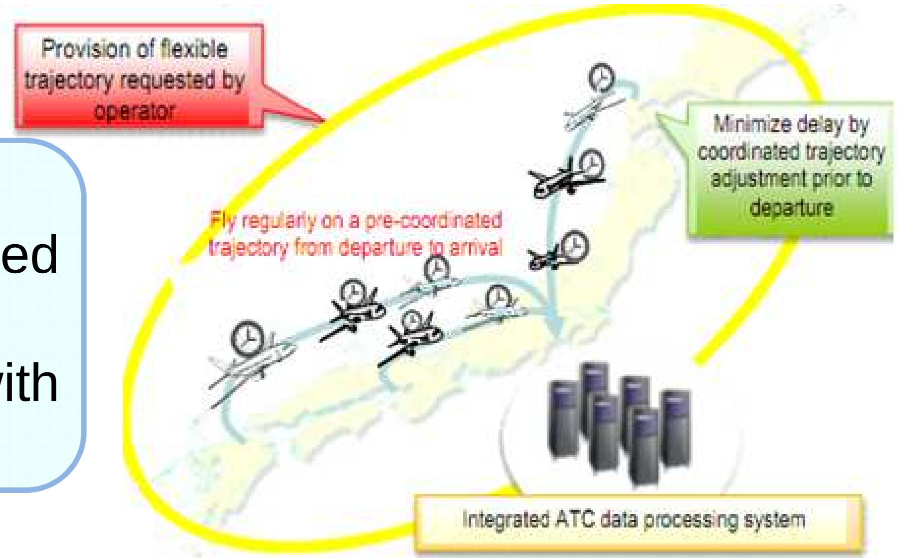
Introduction

Background

Airspace based operation is conventionally used today in Air Traffic Control (ATC).

Disadvantages
- ✓ Bound to perform on the predetermined flight route and altitude
- ✓ Congestion at terminal airspace with inefficient air traffic flow management

Trajectory Based Operations (TBO)



Following two advantages are expected by the realization of TBO.

- ➢ Reduction of fuel consumption, which is beneficial for economy and ecology.
- ➢ Enhancing air traffic capacity at terminal airspace and ground handling capacity.

Introduction

Free Flight which maximizes each aircraft performance is ideal for a futuristic air transportation system.

Free Flight concept was proposed to improve the efficient usage of airspace, providing users the capability of selecting the optimal flight route and airspeed.

- Trajectory optimization is a key technology to realize an ideal air transportation system in the foreseeable future.
- An efficient tool which provides plausible solutions is necessary for the trajectory optimization.

Objectives

- Developing a trajectory optimization tool for a jet passenger aircraft to generate fuel efficient flight trajectories.
- Proposing an effective method which reduces the computational time for trajectory design.

An example of 4D optimal trajectory

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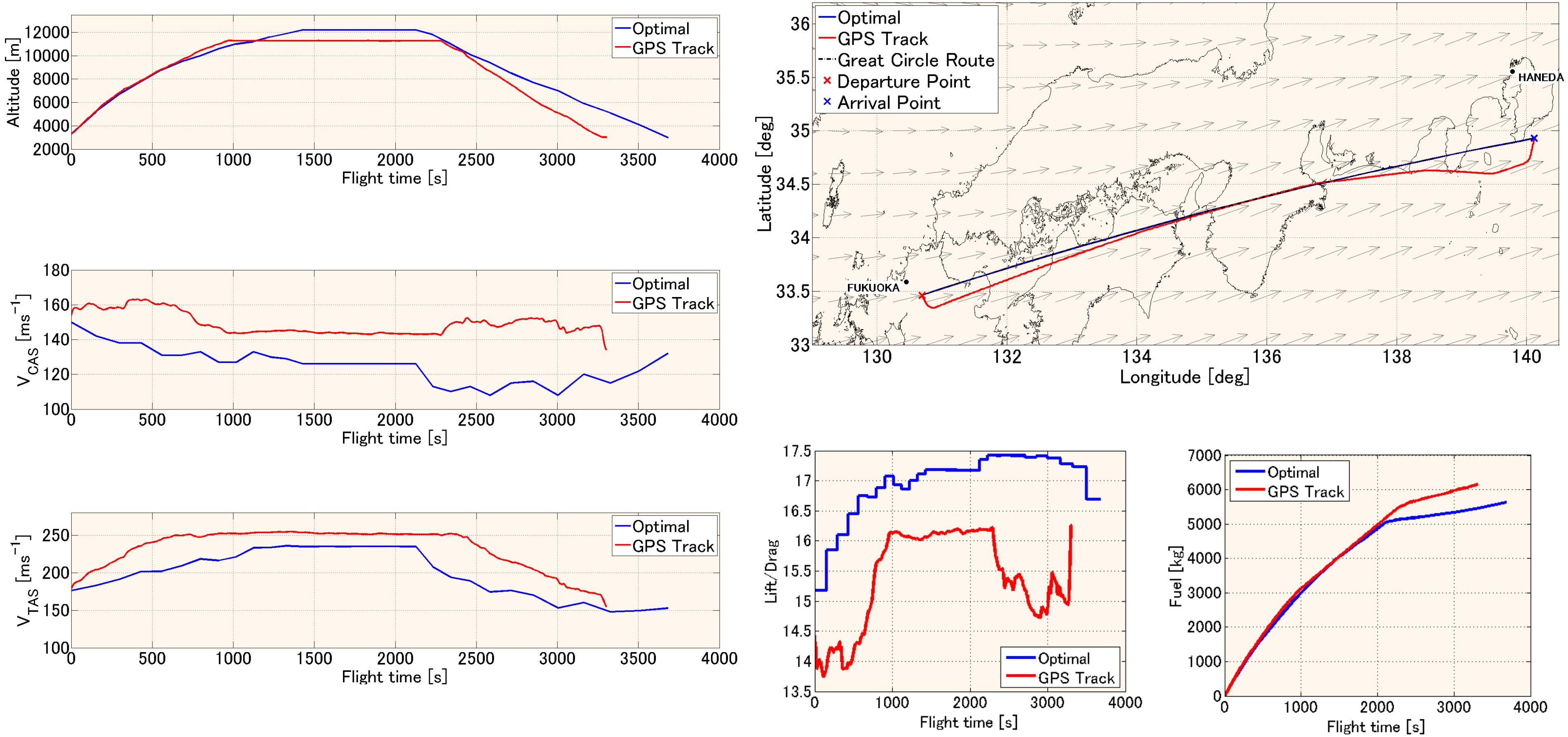
4D Optimal trajectory considering the presence of wind



[EN-41] A Study on Benefits Gained by Flight Trajectory Optimization for Modern Jet Passenger Aircraft
(Conference Room 2, 16:30～)

Optimal control problem

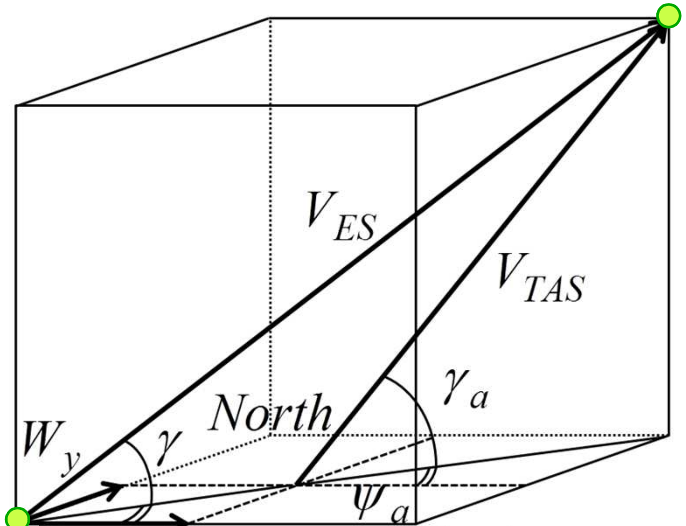
Equations of motion

$$\frac{d\theta}{dt} = \frac{1}{(R_0 + H)\cos\phi}\{V_{TAS}\cos\gamma_a \sin\psi_a + W_x\}$$

$$\frac{d\phi}{dt} = \frac{1}{R_0 + H}\{V_{TAS}\cos\gamma_a \cos\psi_a + W_y\}$$

$$\frac{dH}{dt} = V_{TAS}\sin\gamma_a$$

$$m\frac{dV_{ES}}{dt}\cos(\gamma_a - \gamma)\cos(\psi_a - \psi) = T - D - mg\sin\gamma_a$$



Dynamics of the aircraft is expressed by the time derivative of longitude θ , latitude ϕ , altitude H and earth speed V_{ES} .

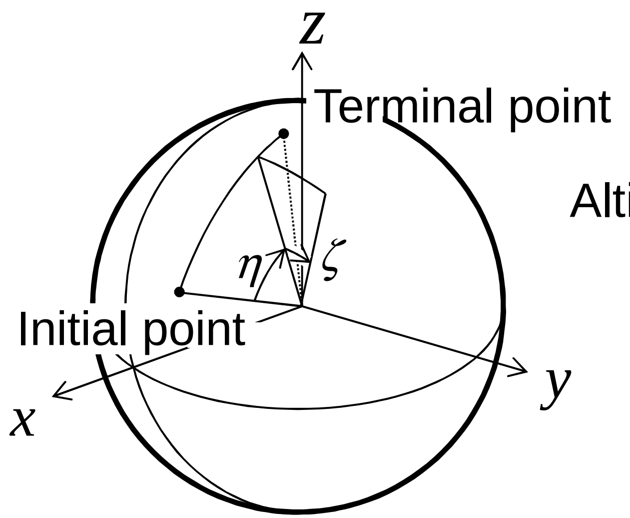
V_{TAS}	:	True air speed
V_{ES}	:	Earth speed
γ	:	path angle
ψ	:	Azimuth angle
W_x	:	Zonal wind
W_y	:	Meridional wind

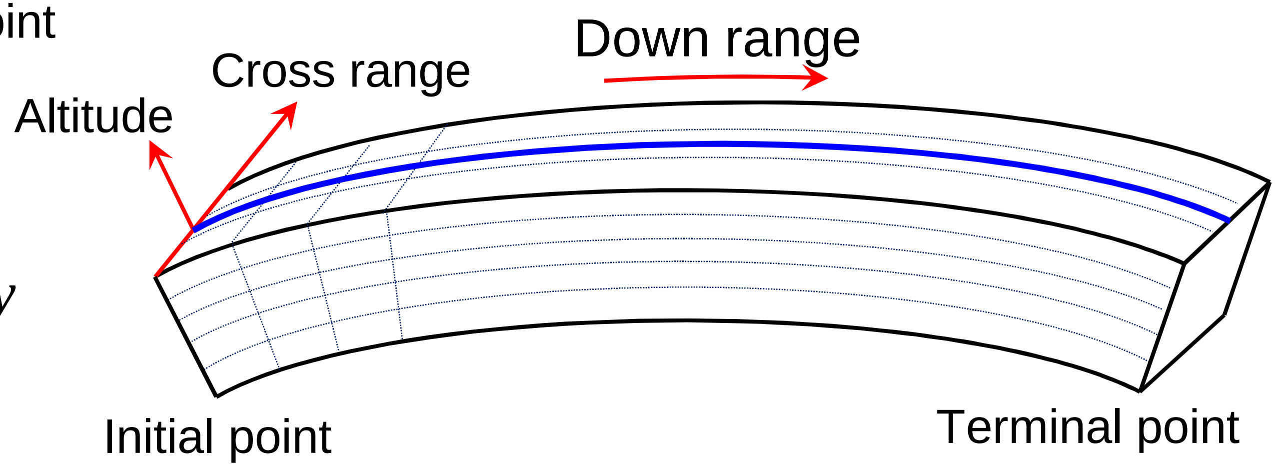
Performance index

$$J = \int_{t_0}^{t_f} FF dt$$ (Total fuel consumption)

Optimal control problem is defined as deriving the optimal trajectory which minimizes the total fuel consumption considering operational limitations.

Grid system





① Calculation grid is defined by using state variables H , V , ς and η.

② Flight trajectory is presented by series of transitions between grid points.

- Control variables γ_a , ψ_a , T are derived between two grid points by solving equations of motion with "difference approximation".

$$T = m\frac{\Delta V_{ES}}{\Delta t}\cos(\gamma_a - \gamma)\cos(\psi_a - \psi) + D + mg\sin\gamma_a$$

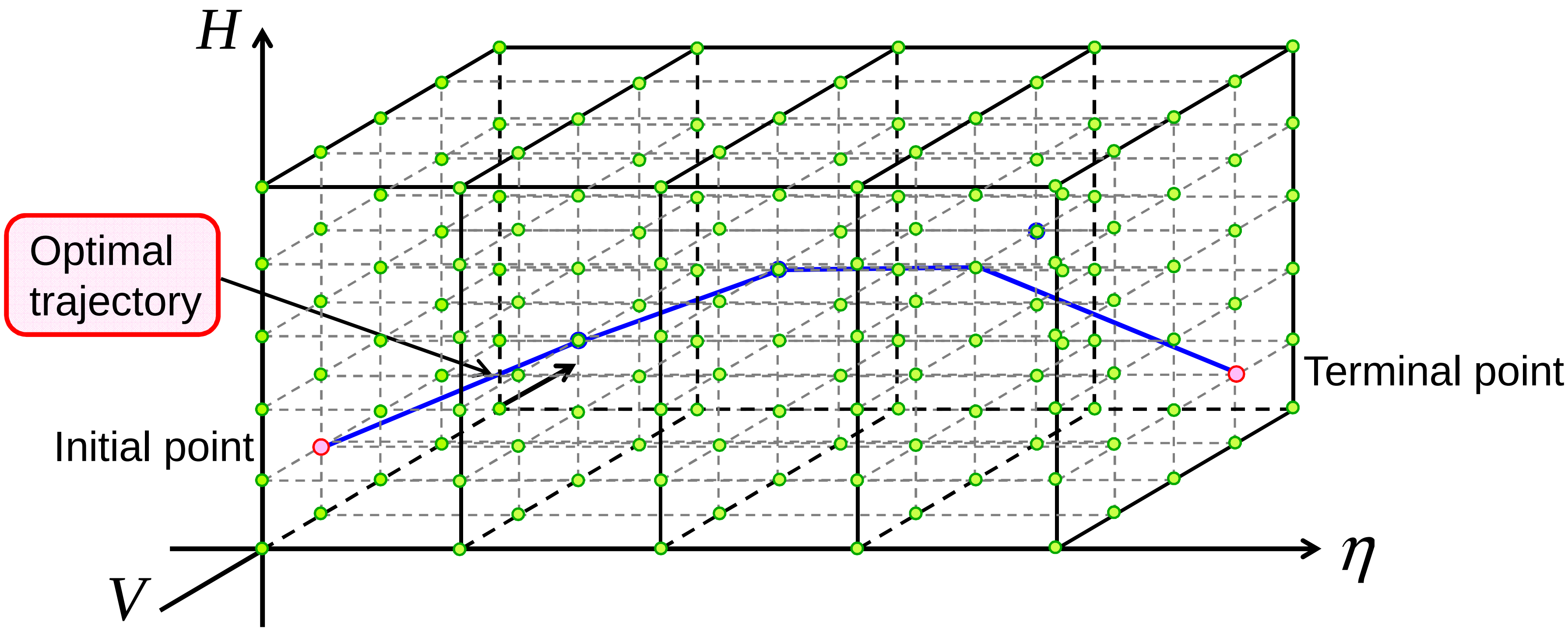
- Fuel consumption is also calculated from the transition between two grid points.

$\Delta Fuel = \Delta J = c_T T \Delta t$ (c_T : Coefficient of specific fuel consumption)

Advantages of DP

- Global optimum
- No iterative calculations
  -Computational time can be estimated in advance.
- Easy to handle inequality constraints on state variables and control variables.
- Easy to adapt for changing design conditions, models and parameters.
- Simple programming

Advantage; Global optimality



State variables: H, V, ς Independent variable: η

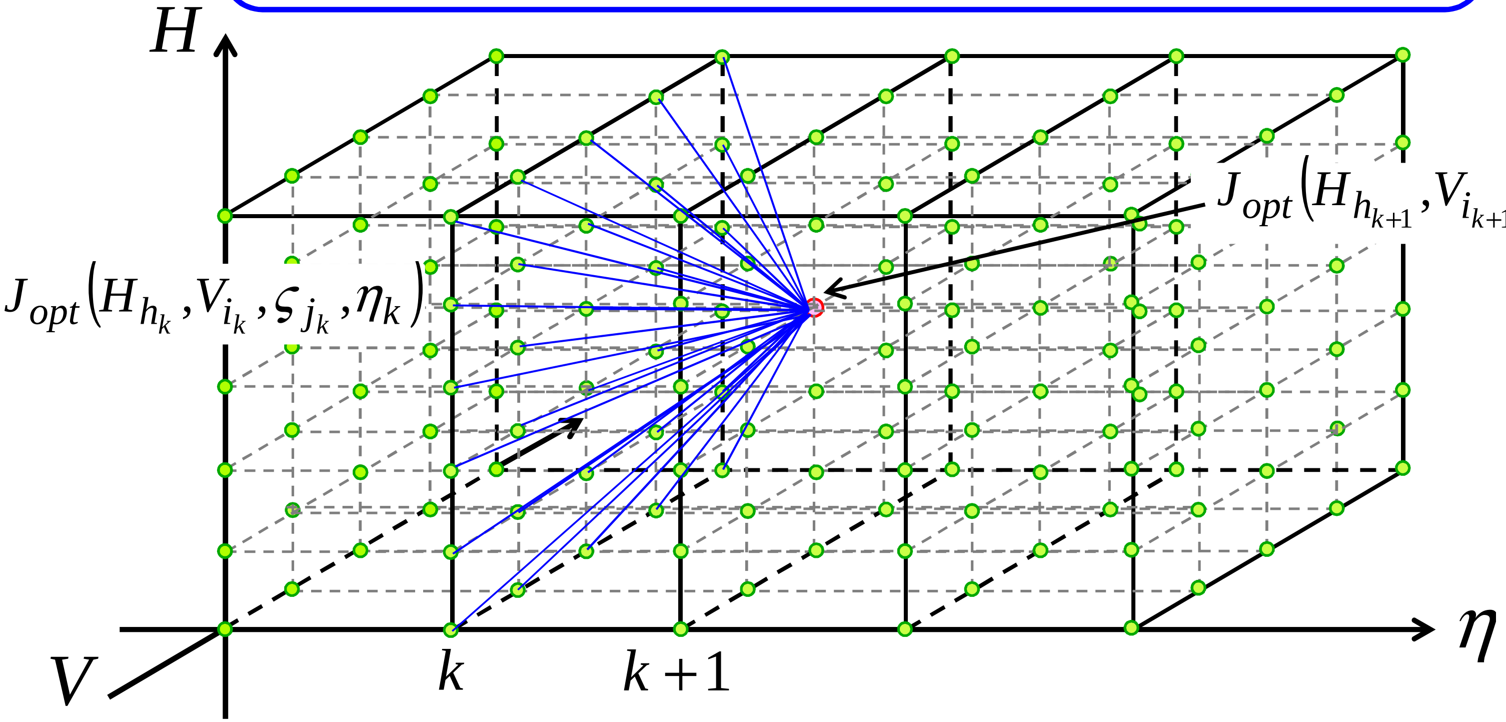
(η :Downrange, ς : Crossrange)

The optimal trajectory obtained is equivalent to that solved as a combinatorial optimization problem for all transitions between the grid points.

Advantage; No iterative calculation

Hamilton-Jacobi-Bellman (HJB) equation

$$-\frac{\partial J_{opt}}{\partial t} = \min_u \left[L(x,u,t) + \frac{\partial J_{opt}}{\partial x} f \right] = \min_u H\left(x, \frac{\partial J_{opt}}{\partial x}, u, t \right)$$



$$J_{opt}\left(H_{h_{k+1}}, V_{i_{k+1}}, \varsigma_{j_{k+1}}, \eta_{k+1}\right) = \min_{k \to k+1} \left[J_{opt}\left(H_{h_k}, V_{i_k}, \varsigma_{j_k}, \eta_k\right) + \Delta J \right]$$

The optimal transition between two neighboring sections of independent variable is governed by the Hamilton-Jacobi-Bellman optimality condition. Therefore, the calculation can proceed one way from the initial or final point.

Advantage; Inequality constraints

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- State variable inequality constraints

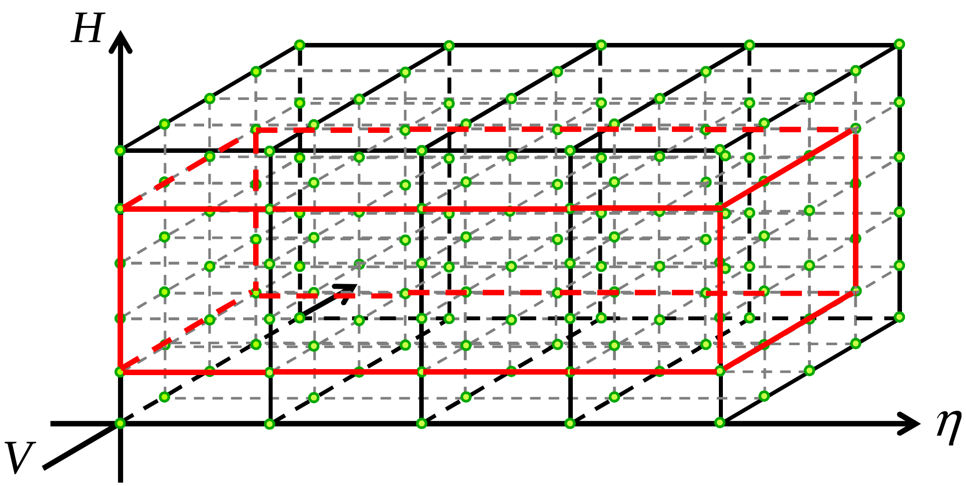
$$F\left(x(t_1), x(t_2), x(t_3), \ldots, x(t_n), x_f\right) \le 0$$

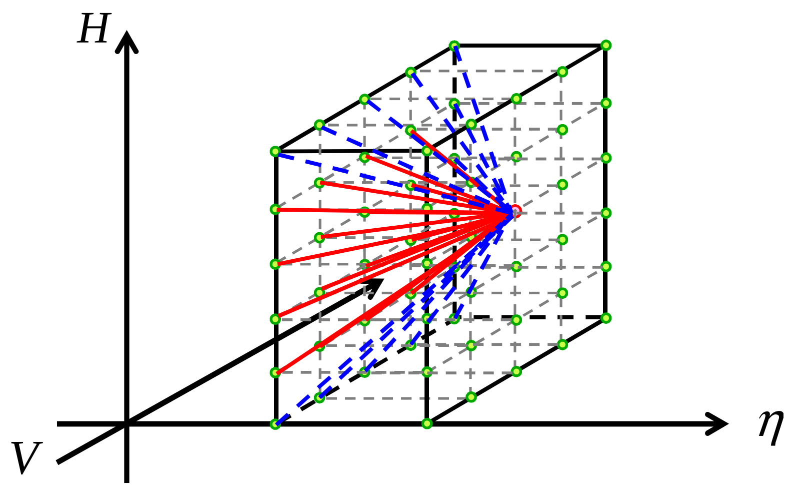
$$H_{\min} \le H \le H_{\max}$$
$$V_{\min} \le V \le V_{\max}$$
$$\varsigma_{\min} \le \varsigma \le \varsigma_{\max}$$
$$\eta_{\min} \le \eta \le \eta_{\max}$$

- Control variable inequality constraints

$$G\left(u(t_1), u(t_2), u(t_3), \ldots, u(t_n), u_f\right) \le 0$$

$$\gamma_{a,\min} \le \gamma_a \le \gamma_{a,\max}$$
$$\psi_{a,\min} \le \psi_a \le \psi_{a,\max}$$
$$T_{\min} \le T \le T_{\max}$$





It's easy to handle inequality constraints on state variable and control variable.

Advantage; Design conditions, models and parameters.

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- Aircraft performance model

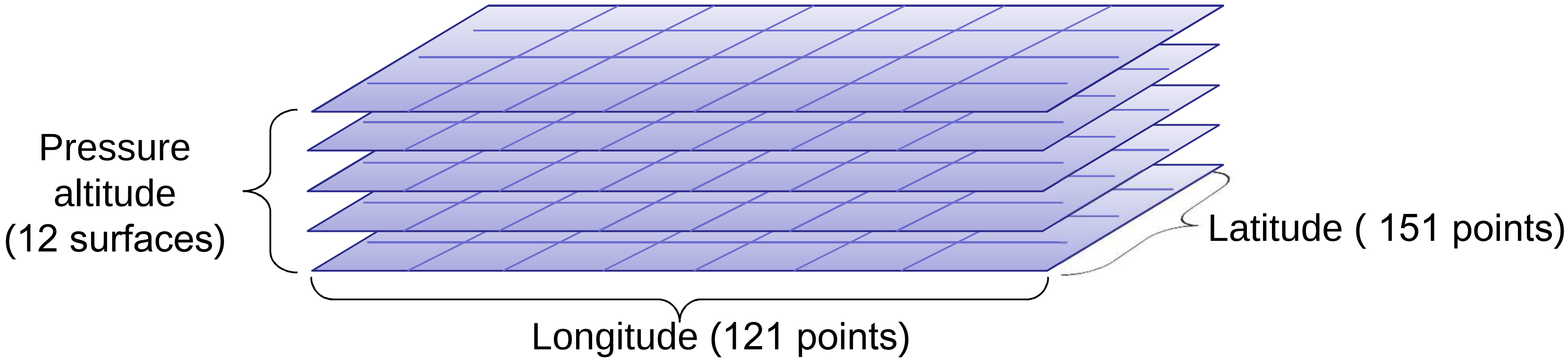
  The performance of an aircraft is calculated by using BADA (Base of Aircraft Data) model which is developed and maintained by EUROCONTROL.

$$FF_{nom} = k_2 c_{f_{cr}} c_{f_1}\left(1+\frac{V_{TAS,kt}}{c_{f_2}}\right)T \qquad FF_{\min} = k_3 c_{f_3}\left(1-\frac{h_{ft}}{c_{f_4}}\right)$$

$$\Rightarrow FF = \max[FF_{nom}, FF_{\min}]$$

- Meteorological data

  Global Spectral Model (GSM) for Japan region provided by Japan Meteorological Agency (JMA) is used.

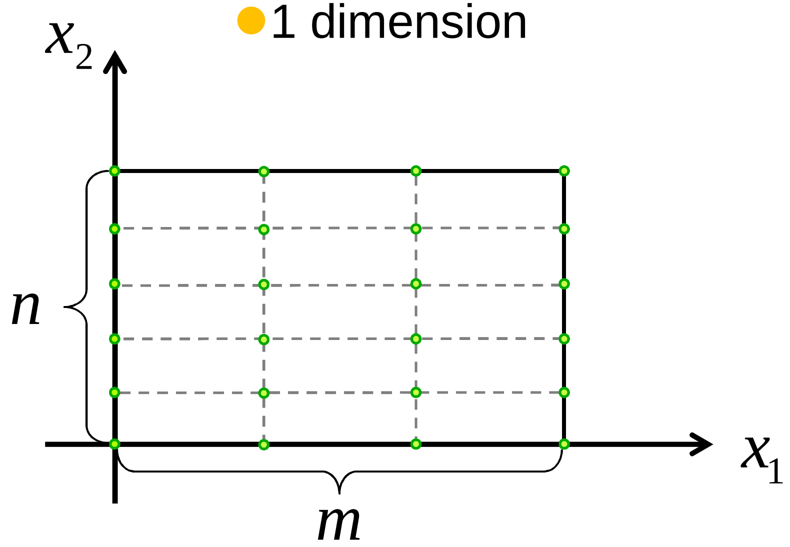


DP has many advantages, however, it has also a major drawback.

Major drawback of DP

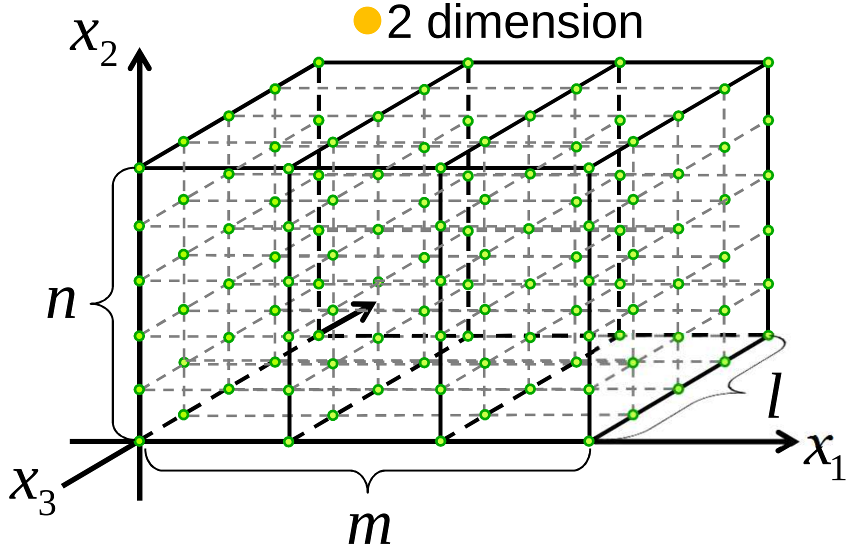
- ✓ Curse of Dimensionality
  - Computational time and required memory increase against the number of grid points.

● 1 dimension



Total amount of computation: $m \times n^2$

● 2 dimension



Total amount of computation: $m \times n^2 \times l^2$

● 3 dimension $m \times n^2 \times l^2 \times o^2$

The total amount of computation increases explosively with the square of state variable grid points number.

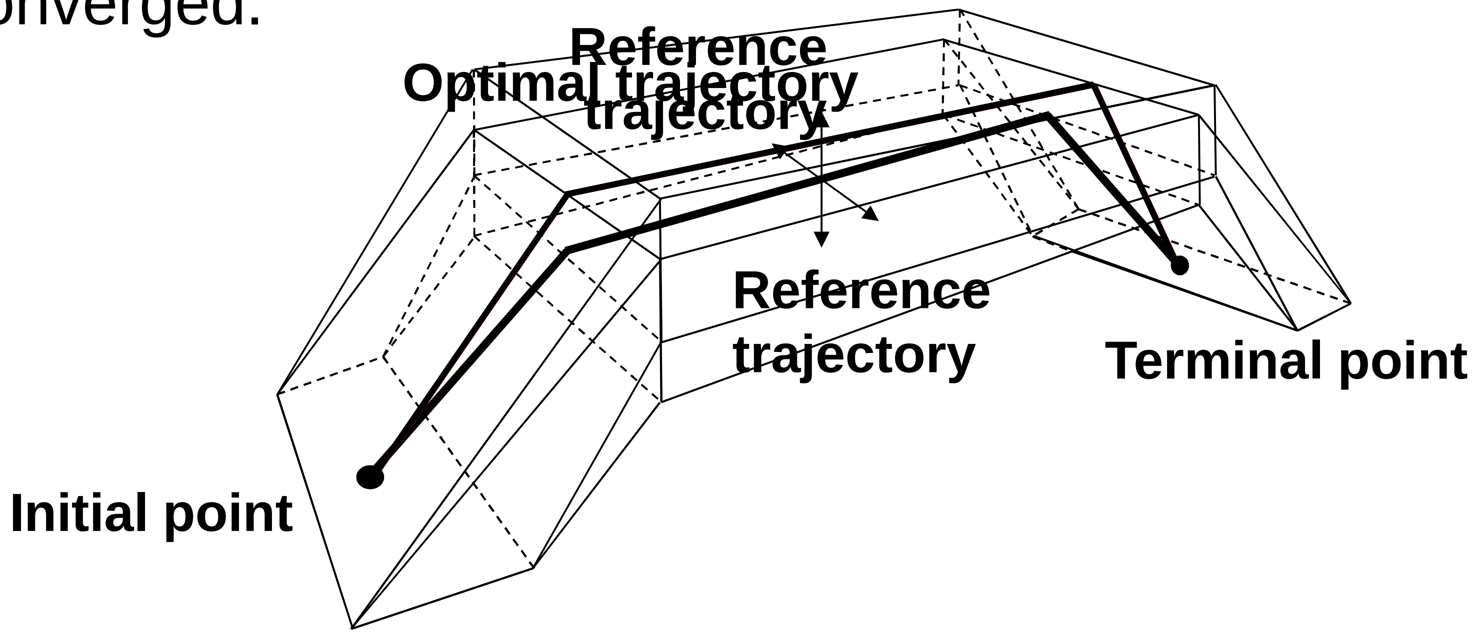
An effective method is proposed to solve this problem.

Moving Search Space Method (MS-DP)

Calculation process

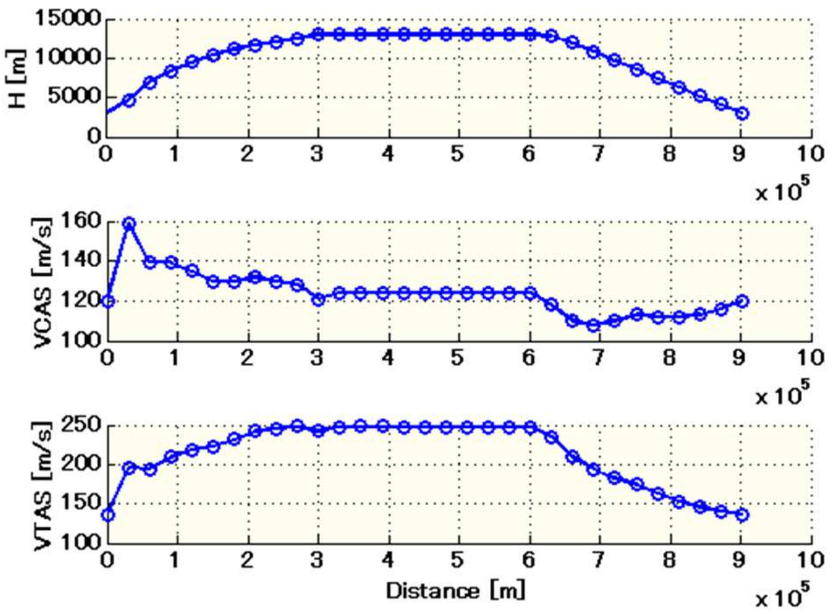
① Partial search space is set around the initial reference trajectory and the optimal trajectory is obtained in the space.

②If the obtained solution is same as the reference trajectory, it is considered as the optimum solution. If not, the calculation process ① is implemented again until the obtained solution is converged.



Calculation example (Full search)

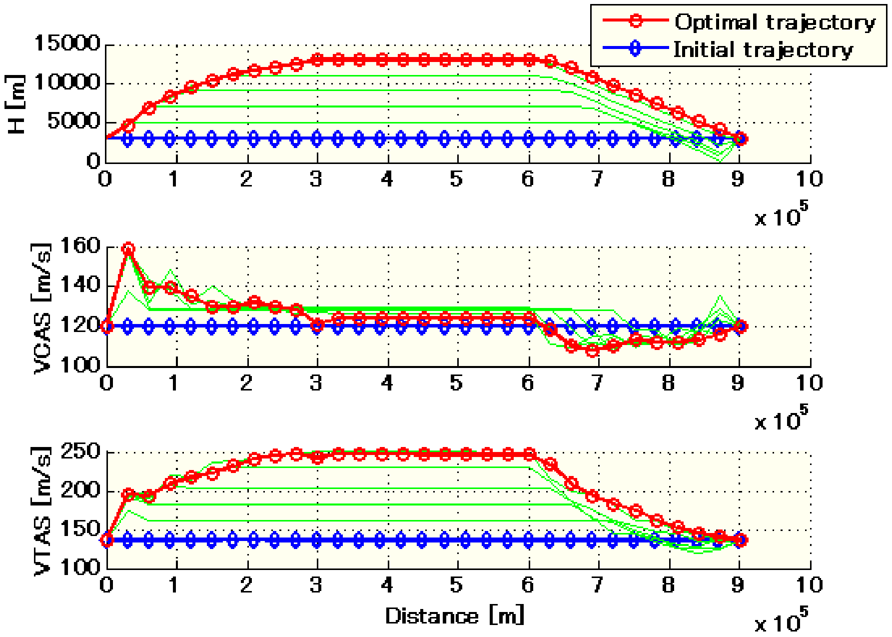
An analysis with no wind conditions is implemented as an example.
At first, the optimal trajectory is derived with all grid points.



	Range	Number of division	Grid resolution
Down range	0~900 [km]	30	30 [km]
Altitude	3000~13,000 [m]	100	100 [m]
Velocity (CAS)	100~160 [m/s]	60	1 [m/s]

Calculation example (MS-DP)

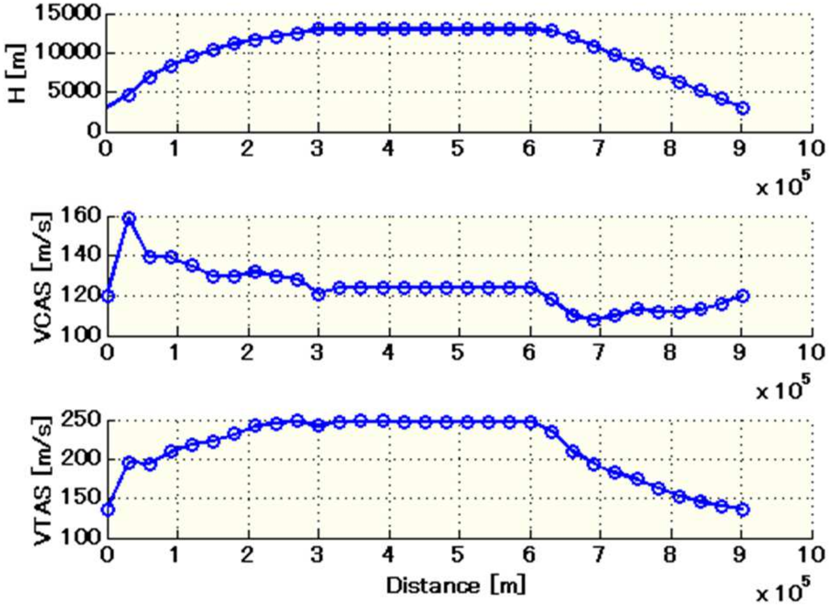
MS-DP method is applied to the same problem in order to reduce the computational time.



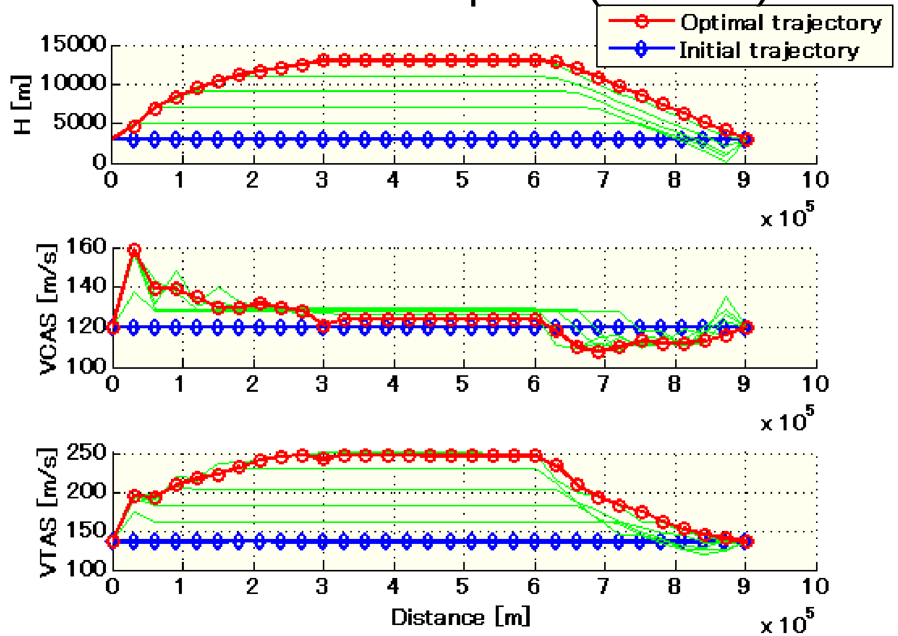
	Grid resolution	Search point around the reference
Altitude	100 [m]	±10
Velocity (CAS)	1 [m/s]	±5

Comparison of results

Full search



Reduced search space (MS-DP)



Exactly the same result as the full search case was obtained by the moving search space method with 18 iterations.

	Full	**Partial**
Computational time	1,321 [sec]	36 [sec]
Total amount of computation	$(101\times61)^2\times31$	$(21\times11)^2\times31\times18$

- Computational time

$$\frac{36}{1321}\times100=2.7[\%]$$

- Total amount of computation

$$\frac{(21\times11)^2\times31\times18}{(101\times61)^2\times31}\times100=2.5[\%]$$

The global optimum has been obtained with MS-DP.

Conclusions

1. A flight trajectory optimization tool useful for future air transportation system research was developed.
2. DP is an easily-handled optimization method due to the simplicity of including inequality constraints and design conditions, and the ability to avoid iterative calculations.
   - It can be considered that DP is suitable for the trajectory optimization of passenger aircraft.
3. DP's major drawback 'The Curse of Dimensionality' could be relaxed to some extent by the proposed moving search space method.
   - It provides the global optimum in most cases by adjusting the amount of limits of the search space.
4. A related study has revealed that the developed DP optimization tool is promising for the design of fuel efficient flight trajectories.