[EN-020] A Queueing Model with Scheduled Arrivals under 4D Trajectory-Based Operations

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Abstract   This paper develops a queueing model for aircraft arrivals at a single server under trajectory-based flight operations, which are expected to prevail in the Next Generation Air Transportation System. Aircraft are assigned scheduled times of arrival at a server, which they meet with some normally distributed stochastic error. A recursive queueing model is formulated, and the Clark approximation method is employed. Exact results are derived for a special case with evenly spaced scheduled times of arrival, and the impact of buffers in the arrival stream is explored.

Keywords aircraft, 4D trajectories, queueing model

1. INTRODUCTION

The nation's air transportation system (NAS) will incur major transformations in the coming years, developing towards the so-called Next Generation Air Transportation System (NextGen). NextGen features a shift from the current static system of routes and sectors to one that is adaptive to weather, traffic, and user preferences. NextGen planners envision a system in which users will exchange trajectory information and supply the Air Navigation Service Provider with greater amounts of information about future traffic demand. This will be used to anticipate and resolve conflicts well in advance, reducing the need for tactical air traffic control. It will also allow controlled times of arrival into busy terminals, weather-impacted airspace, and other bottlenecks. This transformation is expected to greatly reduce human operator workload and significantly increase airport and airspace capacity.

The motivation for this research is the fact that the ability to control and predict 4D aircraft trajectories (4DT) with high precision is a cornerstone of NextGen. 4DT capability, with time being the fourth dimension, is defined as the ability to precisely fly an assigned 3D trajectory while meeting specified times of arrival at certain waypoints [1]. This will allow high density flows that rely on controlled times of arrival for critical resources, including entry and exit to/from airspace regions, taxiways, and runways [1]. Thus, in this research we assume that an aircraft's flight path includes a series of waypoints (that can be either points in the airspace or the runway's threshold) that the aircraft has to cross at a scheduled time. In other words, we assume that under 4DT operations aircraft will be metered at fixes.

However, even with the deployment of the very best 4D trajectory precision and navigation tools, adherence to 4D trajectories — in particular to scheduled arrival times at the fixes — will not be perfect. Sources of imprecision include airframe-to-airframe variation in aerodynamic performance, limitations in wind prediction capability, variations in flight crew technique, and varying degrees of exactitude in navigational performance [1]. As the NAS evolves from its current state to a future condition where location precision is maximized, trajectory precision will vary in response to these factors. It will range from low precision, corresponding to today's operations in the NAS, to high precision, enabled by full deployment and utilization of precision navigation and 4DT trajectory awareness tools. Operations at the two ends of this spectrum can be modeled using stochastic and deterministic queuing models; see [2]. While the models for such cases are well established, it is far more challenging to consider intermediate levels of stochasticity. Such cases must, however, be considered when modeling NextGen since trajectory adherence will be imperfect, but sufficient to invalidate the assumptions of existing stochastic models. Thus, the objective of this paper is to model aircraft operations in NextGen, in a way that accounts explicitly for varying levels of trajectory uncertainty.

Existing stochastic queueing models typically assume that the aircraft arrival process at an airport's terminal airspace area is a non-homogeneous Poisson process [3]. However, for trajectory-based operations in NextGen, the Poisson-arrivals assumption does not capture the concept of metered aircraft operations. Thus, in this paper, we propose a queueing model that can analyze flight delays in

a high-precision trajectory-based operational environment, as is envisioned under NextGen.

Within transportation engineering context, queueing models with scheduled arrivals have been proposed to study port operations. Sabria and Daganzo [4] examine single server queueing systems where customers must be served in an order that is specified by a timetable, i.e. in a First-Scheduled-First-Served (FSFS) order. Each customer has a scheduled time of arrival at the server, where they actually arrive with some stochastic lateness (positive or negative). Exact transient solutions are obtained for the case when the lateness distribution is Gumbel, and service times are deterministic. In the present paper though, stochastic deviations from scheduled times of arrival are assumed to follow a Normal distribution, while the rule of FSFS service is maintained.

In addition to developing a queueing model, we examine the effect of buffer time between scheduled arrivals on system delay. Although any delays due to unpunctual aircraft arrivals can be absorbed in the inserted buffer, the system incurs losses in throughput performance. Therefore two types of inefficiencies can be defined: those due to reduced throughput and those to unpunctual arrivals. While the former can be planned ahead, the latter are unpredictable. Thus, a different cost coefficient can be assigned to each type of inefficiency and we seek to find the optimal amount of buffer time that minimizes the total cost of delay.

The rest of the paper is organized as follows: Section 2 presents the general form of our model and its approximate solution, and report on experiments conducted to assess the accuracy of the model against simulation. In Section 3 we consider a special case, in which aircraft are metered at a constant rate and the separation requirements as well as 4DT precision levels are same among all aircraft. Working with a queueing model in this simplified form enables us to shed light in the issue of finding an optimal metering rate that attains high throughput while keeping delays due to imprecise adherence to 4DT's small. Finally, Section 4 summarizes our main findings and discusses several directions for future research.

2. THE MODEL AND AN APPROXIMATE SOLUTION

2.1 Model Formulation

Our queueing system consists of a single server, which is a fix (either a point in the airspace or a runway's threshold), and of airplanes that must cross this fix. Aircraft are assigned scheduled times of arrival at the fix, and they fly 4D trajectories to arrive at the fix at their scheduled times. However, due to imprecision in trajectory adherence, aircraft's actual time of arrival at the fix has some stochastic deviation from its scheduled arrival time. The sources of imprecision might include airframe-to-airframe variation in aerodynamic performance, limitations in wind prediction capability, variations in flight crew technique, and varying degrees of exactitude in navigational performance [1]. In addition, consecutive aircraft must maintain a minimum headway h, which can vary over pairs of arriving aircraft, for safety reasons. Since air traffic controllers, with guidance from separation rules, decide values for h, we consider it as a deterministic variable in our model. Moreover, we assume that h is the binding constraint among all factors that may affect the minimum required separation between consecutive aircraft.

Following Sabria and Daganzo's approach, each airplane i has an arrival time at the server A_i that consists of a deterministic and a stochastic portion. The deterministic component a_i is the scheduled arrival time at the fix, while the stochastic component is denoted as $\tilde{A}_i$ and represents the lateness (positive or negative) with which the aircraft arrives at the fix, due to imprecision in trajectory adherence. Therefore, we have $A_i = a_i + \tilde{A}_i$.

We assume that deviations $\tilde{A}_i$'s are small enough that serving aircraft on a *First-Scheduled-First-Served* (*FSFS*) order will not significantly increase delays. As an order of magnitude, NextGen planners foresee accuracies of ±10 seconds in aircraft meeting scheduled times of arrival [5]. Under a *FSFS* queue discipline, the actual time airplane i departs from the server, D_i, would be A_i if there were no queue at the server by the time it arrived, or the time the previous scheduled aircraft i-1 crossed the fix plus a minimum required separation headway $h_{i-1,i}$ between the two aircraft. The actual times that aircraft cross the fix under study would then be:

$$D_1 = A_1$$
$$D_i = \max\left(A_i, D_{i-1} + h_{i-1,i}\right), \quad \forall i \geq 2$$

If there were no stochasticity in the system, the deterministic time of departure from the server would be:

$$d_i = \max\left(a_i, d_{i-1} + h_{i-1,i}\right), \quad \forall i \geq 2$$

Accounting for stochasticity, the departure time from the server of airplane i is:

$$D_i = d_i + \tilde{D}_i$$

The distribution of the stochastic component $\tilde{D}_i$ clearly depends on $\tilde{A}_i$, which captures all stochastic effects that cause flight i to arrive at a time other than its scheduled one a_i:

$$\tilde{D}_1 = \tilde{A}_1 \qquad\qquad (1a)$$

$$\tilde{D}_i = \max\left(a_i + \tilde{A}_i, d_{i-1} + \tilde{D}_{i-1} + h_{i-1,i}\right) - d_i, \quad \forall i \ge 2 \qquad (1b)$$

The second pivotal assumption is that the vector of stochastic errors $\tilde{A}_i$ follows a multivariate normal distribution with zero means (without loss of generality), standard deviations σ_i, and a covariance structure Σ: $\tilde{\mathbf{A}} \sim \text{Normal}(\mathbf{0}, \Sigma)$. The normality assumption stems from the observation that the probability distribution for $\tilde{A}_i$ is generated by convolving the individual distributions of low-correlated stochastic factors. It should be emphasized though, that $\tilde{A}_i$'s do not represent factors such as, departure delays, traffic management initiatives, severe weather, or en-route congestion that cause significant amounts of delays; lateness effects due to such factors have already been incorporated in the estimation of scheduled arrival times a_i.

In practice, values for schedule deviation σ_i could be aggregated to represent classes of aircraft that have similar capabilities of adherence to 4D trajectories. For example, one could assume two different values for the standard deviation, σ_A and σ_B, in order to roughly represent aircraft with and without Area Navigation (RNAV) and Required Navigation Performance (RNP) capabilities. Alternatively, one could also differentiate aircraft's ability to precisely fly 4D trajectories according to the en-route weather conditions they encounter.

2.2 Solution with the Clark Approximation Method

In equation (1), for $i=2$ both terms of the $\max(\bullet)$ operator are normally distributed. The *max* operation on normal random variables, in contrast to the *add* operation, does not yield a normal random variable. A well-known result due to Clark [6] derives analytical formulas for the mean and variance of the maximum of two normally distributed random variables. Let X and Y be normally distributed random variables, $X \sim \text{N}(\mu_X, \sigma_X)$ and $Y \sim \text{N}(\mu_Y, \sigma_Y)$, ρ represent the correlation coefficient between X and Y, and Z be the maximum of X and Y, $Z \triangleq \max(X, Y)$. The mean μ_Z and variance σ_Z^2 of Z are then:

$$\mu_Z = \mu_X \Phi(\alpha) + \mu_Y \Phi(-\alpha) + \gamma \varphi(\alpha)$$
$$\sigma_Z^2 = \left(\sigma_X^2 + \mu_X^2\right)\Phi(\alpha) + \left(\sigma_Y^2 + \mu_Y^2\right)\Phi(-\alpha)$$
$$+ \left(\mu_X + \mu_Y\right)\gamma\varphi(\alpha) - \mu_Z^2$$

where

$$\gamma \triangleq \left(\sigma_X^2 + \sigma_Y^2 - 2\rho\sigma_X\sigma_Y\right)^{1/2}$$
$$\alpha \triangleq \frac{\mu_X - \mu_Y}{\gamma}$$
$$\varphi(x) \triangleq \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{x^2}{2}\right)$$
$$\Phi(y) \triangleq \int_{-\infty}^{y} \varphi(x)\,dx$$

The coefficient of linear correlation between Z and a third normal random variable W can also be estimated, given that we know the coefficients of linear correlation between X and W $(\rho_{X,W})$, and between Y and W $(\rho_{Y,W})$:

$$r[W, Z] = \left(\sigma_X \rho_{X,W}\Phi(\alpha) + \sigma_Y \rho_{Y,W}\Phi(-\alpha)\right)/\sigma_Z$$

The above formulas give the exact mean and variance of Z. The approximation is introduced by assuming that Z follows a normal distribution with mean μ_Z and variance σ_Z^2.

In the context of our problem with scheduled aircraft arrivals, the Clark approximation method can be used for all $i \ge 2$ to approximate D_i's as normal random variables, and estimate their mean $E(D_i)$ and variance $Var(D_i)$ in a recursive manner:

$$E(D_i) = a_i \Phi(\alpha_i) + \left[E(D_{i-1}) + h_{i-1,i}\right]\Phi(-\alpha_i) + \gamma_i \varphi(\alpha_i) \quad (2)$$

$$Var(D_i) = \left(\sigma_i^2 + a_i^2\right)\Phi(\alpha_i) +$$
$$+ \left[Var(D_{i-1}) + \left[E(D_{i-1}) + h_{i-1,i}\right]^2\right]\Phi(-\alpha_i) + (3)$$
$$+ \left[a_i + E(D_{i-1}) + h_{i-1,i}\right]\gamma_i \varphi(\alpha_i) - \left[E(D_i)\right]^2$$

$$r[A_{i+1}, D_i] = [\sigma_i \cdot \rho_1 \Phi(\alpha_i) +$$
$$+ \sqrt{Var(D_{i-1})} \cdot \rho_2 \cdot \Phi(-\alpha_i)] / \sqrt{Var(D_i)} \qquad (4)$$

where

$$\gamma_i = \left(\sigma_i^2 + Var(D_{i-1}) - 2 \cdot \rho \cdot \sigma_i \cdot \sqrt{Var(D_{i-1})}\right)^{1/2} \qquad (5)$$

$$\alpha_i = \frac{a_i - E(D_{i-1}) - h_{i-1,i}}{\gamma_i} \quad (6)$$

and at each step i
$$\rho = r[A_i, D_{i-1}], \quad \rho_1 = r[A_{i+1}, A_i], \quad \rho_2 = r[A_{i+1}, D_{i-1}].$$

Note that $r[A_i, D_{i-1}]$ and $r[A_{i+1}, D_{i-1}]$ are obtained through equation (4) in previous iterations. Effectively, the method is implemented by estimating at each step k

$r\left[A_i, D_k\right]$ for all $i > k$. Moreover, $r\left[A_{i+1}, A_i\right]$ is considered as input from covariance matrix Σ. Equations (2)-(6) are easy to program and they are computationally efficient. Finally, for a stream of N flights scheduled for to arrive at a fix, the total expected delay is defined as:

$$E\left[W_N\right] \triangleq \left[\sum_{i=1}^{N} E\left(D_i\right) - a_i\right]$$

This completes the formulation of our queueing model. In summary, the model requires as inputs a schedule of arrival times a_i, a capacity profile expressed in terms of time separation headways $h_{i-1,i}$, and a covariance matrix of trajectory adherence errors $\sigma_{i,j}$. These, coupled with the assumption that σ_i's are small enough to allow a first-scheduled, first-served policy, enable the estimation of expected flight delays through Clark's approximation method.

2.3 Approximation Error

Although the maximum Z of two normal random variables X and Y is not normally distributed, our model is based on approximating Z with a normal random variable. In particular, in estimating $D_i = \max\left(A_i, D_{i-1} + h_{i-1,i}\right)$ it is assumed that D_{i-1} is normally distributed. That enables the estimation of the mean and variance of D_i, which is then also approximated as a normal random variable. However, each pair-wise operation introduces some error that is propagated and might affect the accuracy of our estimates. For a thorough analysis on this topic, see Sinha et al. [7] and Horowitz et al. [8].

To test the accuracy of the Clark Approximation Method in the context of our analysis, several operational scenarios were considered. The estimates from the analytical queueing model were then compared against the average of 10^4 Monte Carlo simulation runs, which is considered as ground truth.

Each operational scenario was formulated as follows: a total of 120 aircraft must cross a fix, and the minimum required separation between any two successive aircraft is

set to $h_{i-1,i} = 30, 60,$ or 90 seconds. Each aircraft is assigned a scheduled time of arrival at the server $a_i = a_{i-1} + h_{i-1,i} + b$, where b denotes a buffer time inserted. Aircraft arrive at the server with some imprecision that follows a normal distribution and has a standard deviation σ. Zero covariance was assumed across the aircraft arrival times at the server A_i. A total of 90 scenarios were examined:

- 10 different sequences of $h_{i-1,i}$ (each sequence has an equal mix of 30, 60, and 90 seconds)
- b = 0, 10, and 20 seconds (held constant within each sequence)
- σ = 10 seconds (uniform across all aircraft), 30 seconds (uniform across all aircraft), and an equal mix of both.

Three metrics for the approximation method accuracy were considered:

1. Percentage Error in total Delay % (PE):
$$\frac{E\left[W_N\right]^{appr} - E\left[W_N\right]^{sim}}{E\left[W_N\right]^{sim}} \cdot 100$$

2. Absolute Error in Total Delay (AE):
$$\left|E\left[W_N\right]^{appr} - E\left[W_N\right]^{sim}\right|$$

3. Flight Departure Time Mean Absolute Deviation (MAD): $\dfrac{\sum_{i=1}^{N}\left|D_i^{appr} - D_i^{sim}\right|}{N}$

The first two metrics evaluate the accuracy of the approximation method in estimating the expected total aircraft delay against assigned scheduled times of arrival. The third metric provides with a measure of the error in predicted outcomes for individual flights.

The results are presented in Table 1. Each entry in the table represents the average value across the ten scenarios of different $h_{i-1,i}$ sequences. In all cases, the Total Delay PE metric indicates that the approximation method is within -8% accuracy in estimating the total delay in the system, as compared to simulation. Moreover, the absolute

Table 1 Results of approximation accuracy tests

	Buffer = 0 (sec)			Buffer = 10 (sec)			Buffer = 20 (sec)		
	PE	AE (sec)	MAD (sec)	PE	AE (sec)	MAD (sec)	PE	AE (sec)	MAD (sec)
$\sigma = 10$	-0.62%	13.78	0.14	-3.26%	9.17	0.09	-3.93%	2.97	0.08
$\sigma = 30$	-0.49%	36.50	0.35	-1.69%	40.92	0.35	-2.41%	31.17	0.31
Mixed	-1.52%	97.26	0.89	-5.74%	79.53	0.65	-7.70%	54.07	0.44

error in estimating total delay does never exceed 1.3 minutes. Also, the MAD metric indicates that the approximation method estimates the expected delay of each aircraft with accuracy better than 1 second, on average. The accuracy of the method slightly decreases when the fleet contains aircraft with different navigation capabilities. This must be due to heterogeneity in the variance of the normal distributions for A_i that enters in the *max* operator in each step of the recursion.

In summary, these experimental results indicate that our proposed model accurately predicts operational consequences of metered operations with good but imperfect 4DT adherence, as might be expected in NextGen.

3. SIMPLIFIED MODEL

3.1 Formulation
We consider the case where average demand for service exceeds capacity over a considerable period of time, and aircraft are metered at the server (e.g. a point in the airspace, or a runway's threshold) at a constant metered headway of a time units. To maximize throughput and minimize expected delay, a should be set to the minimum headway, h. It may, however, better to set a to be slightly greater than h, for at least two reasons. First, while increasing a increases delay, it also make the delay more predictable. Airlines value such predictability because it enables them to better plan terminal operations. Second, the delays that result from setting a above h can be absorbed in a more efficient manner, by increasing time at higher altitudes and reducing speed slightly over a longer distance rather than dramatically as the aircraft approaches the metering fix. For these reasons, aircraft operators may be willing to accept an increase in overall delay in exchange for a reduction in the amount of stochastic delay that results from imperfect trajectory adherence. In this section we employ our model to quantify this trade-off.

We focus on a single fix, and make the following assumptions:

- The required minimum time headway h between successive aircraft is constant and deterministic.
- Flight arrivals are uniformly scheduled during the analysis period, with metering headway a.
- Stochastic errors $\tilde{A}_i$ are *i.i.d.* random variables with zero mean and standard deviation σ.

In a fully deterministic environment, there will be no delays if the separation headway a is greater than the minimum headway h. However, if an airplane arrives later than its scheduled arrival time, then it might cause delay to airplanes upstream of it. We are interested in quantifying the expected amount of delay that each airplane in the stream incurs.

Since $a \geq h$, the deterministic component of departure times from the server would be $d_i = ia$, and thus Eq. (1) becomes:

$$\tilde{D}_1 = \tilde{A}_1 \qquad (7a)$$

$$\tilde{D}_i = \max\left(\tilde{A}_i, \tilde{D}_{i-1} + h - a\right), \quad \forall i \geq 2 \qquad (7b)$$

where $\tilde{A}_i \sim \mathrm{N}(0, \sigma), \ \forall i \geq 1$.

We are interested in deriving an analytical expression for $E(\tilde{D}_i)$, but this is a rather difficult task. Instead, we define as $\tilde{Z}_i \triangleq \max\left(\tilde{A}_i^*, \tilde{Z}_{i-1} - \Delta\right)$ where $\tilde{A}_i^* \sim \mathrm{N}(0,1)$, and resort to dimensional analysis by employing the following proposition:

Proposition 1. *For a given level of the relative buffer* $\Delta \triangleq (a-h)/\sigma$, *the sequence of random variables* $\tilde{D}_i$ *is proportional to the sequence of random variables* $\tilde{Z}_i$ *by* σ *standard deviations:*

$$\tilde{D}_i = \sigma \cdot \tilde{Z}_i \qquad (8)$$

Proof. Assume that for some $i \geq 2$, $\tilde{D}_{i-1} = \sigma \cdot \tilde{Z}_{i-1}$. Then we have:

$$\tilde{D}_i = \max\left(\tilde{A}_i, \tilde{D}_{i-1} - (a-h)\right)$$
$$= \max\left(\sigma \cdot \tilde{A}_i^*, \sigma \cdot \tilde{Z}_{i-1} - \sigma \cdot \Delta\right) = \sigma \cdot \tilde{Z}_i$$

But note that for $i = 2$:

$$\tilde{D}_2 = \max\left(\tilde{A}_2, \tilde{D}_1 - (a-h)\right)$$
$$= \max\left(\sigma \cdot \tilde{A}_2^*, \sigma \cdot \tilde{A}_1^* - \sigma \cdot \Delta\right) = \sigma \cdot \tilde{Z}_2$$

Hence, for $i \geq 3$ the result follows by induction. $\quad \therefore$

Therefore, the problem of estimating $E(\tilde{D}_n)$ is reduced to estimating $E(\tilde{Z}_n)$. To estimate the mean of $\tilde{Z}_n$ we first derive its cumulative distribution function:

$$F_{\tilde{Z}_n}(x) = \Phi(x) \cdot \Phi(x + \Delta) \cdot \ldots \cdot \Phi(x + (n-1)\Delta)$$

The probability distribution function of $\tilde{Z}_n$ is therefore:

$$f_{\tilde{Z}_n}(x) = \sum_{i=1}^{n} \prod_{k=1}^{n} \Phi\left(x + (k-1)\Delta\right)^{(\omega_k)}, \quad \begin{array}{l} \omega_k = 1 \quad for \ \ i = k \\ \omega_k = 0 \quad for \ \ i \neq k \end{array}$$

where the term (ω_k) indicates a derivative. The mean of $\tilde{Z}_n$ is therefore:
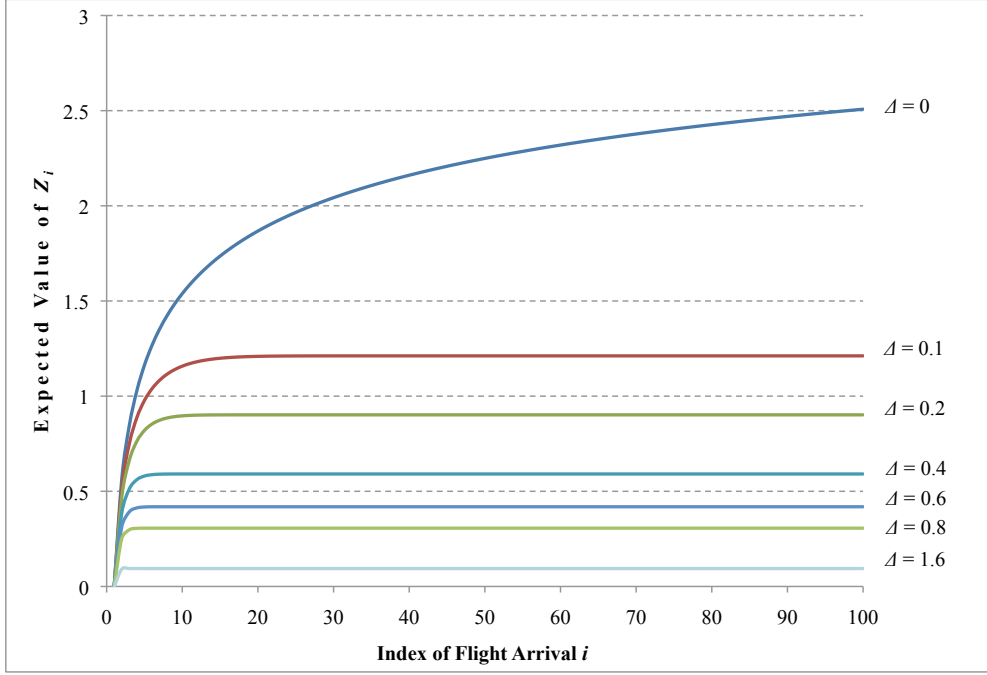
Figure 1 Expected delay $E\left(\tilde{Z}_i\right)$ for several values of Δ

$$E\left[\tilde{Z}_n\right] = \int_{-\infty}^{+\infty} x \cdot \sum_{i=1}^{n} \prod_{k=1}^{n} \Phi\left(x + (k-1)\Delta\right)^{(\omega_k)} dx,$$

where $\qquad\qquad\qquad\qquad\qquad$ (9)

$$\omega_k = 1 \quad for \quad i = k$$
$$\omega_k = 0 \quad for \quad i \neq k$$

Therefore, it suffices to consider cases for Δ and compute $E\left(\tilde{Z}_i\right)$ through numerical integration; Fig. 1 depicts several such curves for $E\left(\tilde{Z}_i\right)$. Thus, for any value of σ, a and h, values for $E\left(\tilde{D}_i\right)$ can be computed by selecting the appropriate curve from Fig. 1 and multiplying $E\left(\tilde{Z}_i\right)$ by σ. In this way, instead of an analytical formula, values for $E\left(\tilde{D}_i\right)$ can be estimated through a lookup table and with the same level of accuracy. Note that the system eventually reaches a steady state (except for the case $\Delta = 0$), after which each flight incurs the same amount of expected delay $E\left(\tilde{Z}_i\right)\sigma$.

Finally, for a stream of N flights scheduled for landing and a given value a–h the total expected delay is equal to:

$$E\left[W_N\right] = \left[\sum_{i=1}^{n} E\left(D_i - ia\right)\right] = \left[\sum_{i=1}^{n} E\left(\tilde{Z}_i\right)\right]\sigma \quad (10)$$

Therefore, the expected total delay of the system is proportional to the standard deviation σ of the stochastic error $\tilde{A}_i$. We can thus plot the expected total delay $E\left[W_N\right]$ against the number of flights N in the arrival stream, for the case when $\sigma = 1$ (see Fig. 2). Observe that for lower levels of the relative buffer Δ, the system incurs higher stochastic delay due to its reduced capacity for absorbing stochastic deviations from schedule.

3.2 Trade-offs between deterministic and stochastic delay

After developing methods for estimating each flight's expected stochastic delay, we proceed to analyzing the trade-off between stochastic delay and throughput. In the context of our analysis, the maximum departure rate from the fix that can be attained is constrained by aircraft minimum separation requirements and is equal to $1/h$. Thus, in order to achieve maximum throughput from the server, one would attempt to meter aircraft at the fix at a headway $a = h$. However, that will result in maximum stochastic delay, as can be inferred from Fig. 1 for $\Delta = 0$. By metering at a headway $a > h$ we decrease the expected amount of stochastic delay, since Δ increases with a, but at the same time we incur increases in deterministic queueing delay. A graphical illustration through a queueing diagram is presented in Fig. 3.
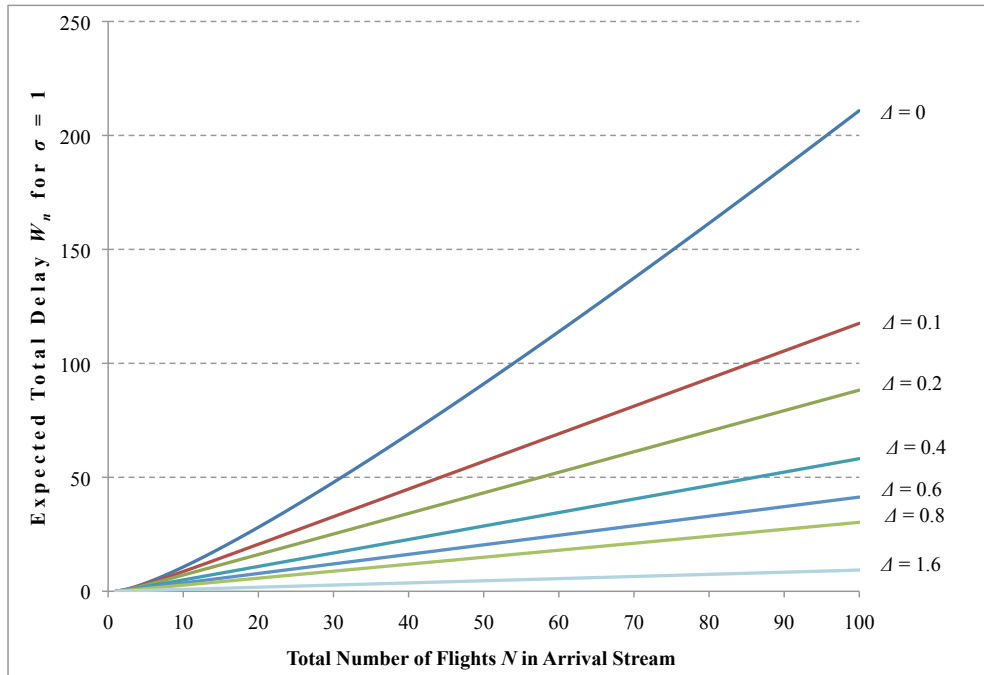
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Figure 2 Expected Total Delay $E\big[W_N\big]$ for several values of Δ

For a surge of N aircraft arrivals, the expected loss from those two types of delay can be expressed as:

$$\mathrm{E}\big[L\big] = 1/2 \cdot N \cdot (N \cdot a - N \cdot h) + \beta \cdot \sum_{i=1}^{N} \mathrm{E}\big[\tilde{Z}_i\big] \cdot \sigma$$

or
$$\mathrm{E}\big[L\big] = \left(1/2 \cdot N^2 \cdot \Delta + \beta \cdot \sum_{i=1}^{N} \mathrm{E}\big[\tilde{Z}_i\big]\right) \cdot \sigma \qquad (15)$$

The coefficient β in the above relationship is the relative cost of stochastic delay over the delay due to reduced throughput. As noted earlier, delays due to reduced metering rate can be planned for well in advance, taken at a higher altitude, in less busy airspace, and with relatively small speed reductions. As an example, we consider two values for the relative cost of stochastic over deterministic delay, namely $\beta = 3$ and $\beta = 10$, as well as two surges of aircraft arrivals, $N = 50$ and $N = 100$. For each case, the
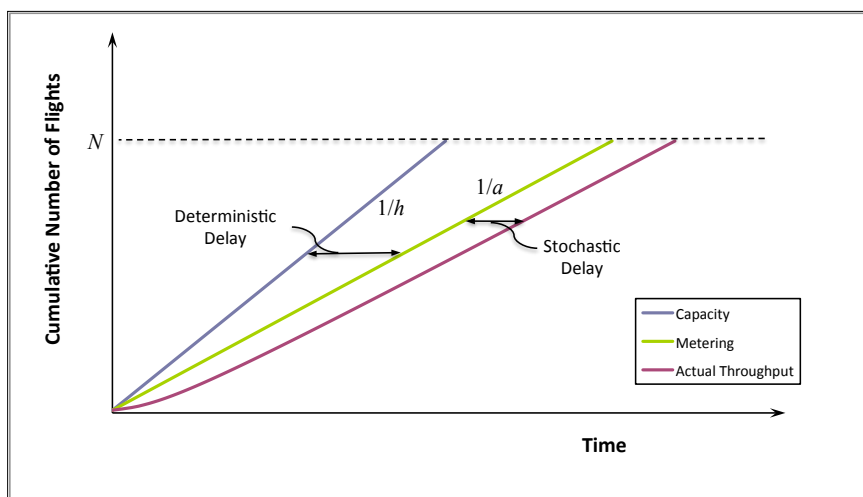

Figure 3 Queueing diagram with deterministic and stochastic delays

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normalized (after setting $\sigma = 1$) expected loss $E\left[L^*\right]$ as a function of Δ is plotted in Fig. 4.

The values of relative buffer that minimize $E\left[L^*\right]$, Δ^*, range between 0.01 and 0.16. Even a value of $\Delta^* = 0.16$ translates to a buffer of 3 seconds between consecutive aircraft for a scenario where $\sigma = 20$ seconds. Also, observe that for constant N, Δ^* increases with β. Therefore, as the unit cost of stochastic delay increases, a larger buffer is required to achieve minimum losses. On the other hand, for constant β, Δ^* decreases with N, indicating that the loss from stochastic delays increases at a lower rate than the loss from deterministic delays, as the surge of aircraft becomes larger. That is expected since deterministic delays increase with N^2 (see Equation 15), while stochastic delays increase linearly with N as it can be observed in Fig. 2.

4. CONCLUSIONS

In this paper a queueing system with a single server under 4D trajectory-based aircraft operations is examined. Aircraft are assigned scheduled times of arrival at a fix, which they meet with some stochastic error. A normal distribution was assumed for the error, and aircraft enter service according to a First-Scheduled-First-Served queue discipline. A recursive queueing model was formulated, and the Clark approximation method was employed to analytically estimate the mean and variance of aircraft delays. The accuracy of the approximated method was validated through simulation experiments, which indicated sufficient accuracy of the Clark method in estimating total system delays.

Next, a simplified situation, with homoskedastic model parameters, was examined in order to provide insights into the queueing system. The impact of buffers in the arrival stream was investigated, and was found that minimum losses from deterministic and stochastic delays are attained when the added buffer is small as compared to the level of precision σ.

Extending this research will relax the FSFS rule of service and allow aircraft to re-sequence themselves as they approach the server. It will also model explicitly the Runway Occupancy Time of an aircraft as a separate random variable, which is now embedded in the deterministic minimum required headway h. Moreover, a distribution other than normal will be considered for the unimpeded arrival times A_i. Finally, an extension to the single server case presented in this paper could be a scenario with a network of servers.
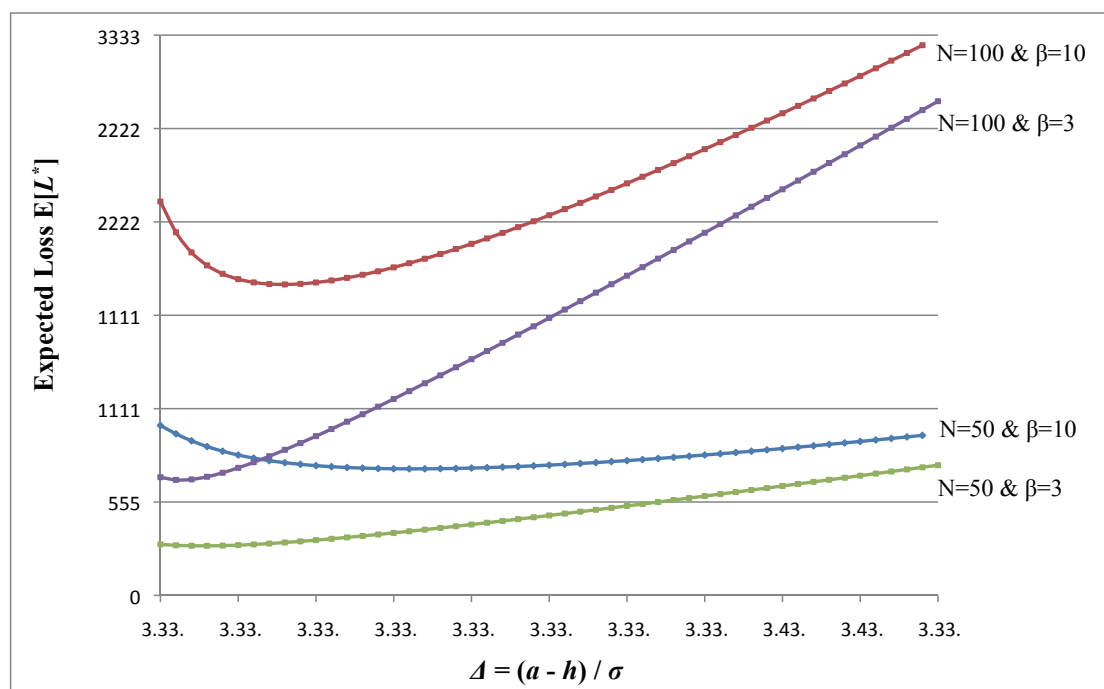
5. ACKNOWLEDGMENTS

Figure 4 Expected loss from deterministic and stochastic delays as a function of Δ

6. REFERENCES

[1] Joint Planning and Development Office, "Concept of Operations for the Next Generation Air Transportation System," version 3.0, 2007.

[2] Hansen, M., Nikoleris, T., Lovell, D., Vlachou, K., and Odoni, A., "Use of Queuing Models to Estimate Delay Savings from 4D Trajectory Precision," Eighth USA/Europe Air Traffic Management Research and Development Seminar, Napa, CA, 2009.

[3] B. O. Koopman, "Air-Terminal Queues under Time-Dependent Conditions," Operations Research, vol. 20, (no. 6), 1972, pp. 1089-1114.

[4] Sabria, F., and Daganzo, C. F., "Approximate Expressions for Queueing Systems with Scheduled Arrivals and an Established Service Order," Transportation Science, vol. 23, (no. 3), 1989, pp. 159-165.

[5] Swenson, H., R. Barhydt, and M. Landis, "Next Generation Air Transportation System (NGATS) Air Traffic Management (ATM)-Airspace Project," Reference Material. External release version, NASA Ames Research Center, 2006.

[6] C.E. Clark, "The Greatest of a Finite Set of Random Variables," Operations Research, vol. 9, (no. 2), 1961, pp. 145-162.

[7] Sinha, D., Zhou, H., and Shenoy, N., "Advances in the Computation of the Maximum of a Set of Random Variables," 7th International Symposium on Quality Electronic Design, San Jose, CA, 2006.

[8] Horowitz, J., Sparmann, J., and Daganzo, C. F., "An Investigation of the Accuracy of the Clark Approximation for the Multinomial Probit Model," Transportation Science, vol. 16, (no. 3), 1982, pp. 382-401.

7. COPYRIGHT