

# [EN-A-055] Cost Index Estimation via Optimization based Trajectory Prediction

(EIWAC 2017)

<sup>+</sup>A. Harada\*, T. Ezaki\*\*, T. Wakayama\*\* and K. Oka\*

\*School of Systems Engineering, Kochi University of Technology  
Kami-shi, Kochi, Japan  
[harada.akinori | oka.koichi]@kochi-tech.ac.jp

\*\*Intelligent Mechanical Systems Engineering Course, Kochi University of Technology  
Kami-shi, Kochi, Japan  
[205034a | 205063d]@gs.kochi-tech.ac.jp

**Abstract:** This paper aims to demonstrate a methodology for estimating CI values of actual flights by a newly-developed trajectory prediction tool. In the future arrival management system, it is considered that the trajectory prediction tool which predicts accurate arrival time will become a key component, i.e., the trajectory must be predicted accurately with the given CI value, flight route and initial mass of aircraft. To ensure validity of the tool's function of arrival time prediction, the actual CI value is estimated using the given radar track and ground speed data. In the trajectory prediction process, flight trajectory optimization is performed by dynamic programming for a set of CI values, and the optimal trajectory closest to the actual one is selected as the predicted trajectory. The base of aircraft data model and Japan Meteorological Agency's Numerical Weather Prediction data are used in the optimization calculation as the realistic aircraft performance model and meteorological data. This trajectory prediction tool assumes to reproduce the trajectory generated by Flight Management System; therefore, the analysis object is limited in the scheduled flights which fly on a RNAV route with constant cruise altitude. Reasonability of the estimated CI values is confirmed by general information listed in aircraft companies' technical document.

**Keywords:** Cost Index, Trajectory prediction, Trajectory optimization, Flight Management System

## Nomenclature

$a$	: weighting parameter for time adjustment
$D$	: Drag
$g$	: gravity acceleration
$H$	: Pressure altitude
$J$	: Performance index / objective function
$L$	: Lift
$m$	: Aircraft mass
$t$	: Time
$T$	: Thrust
$V$	: Velocity
$x$	: Flight distance along track
$\gamma$	: Flight path climb angle
$\mu$	: Fuel flow
Subscript	
$a$	: Air vector component
$W$	: Wind

## 1. INTRODUCTION

The world air traffic has been growing over the past decades. According to the Boeing's current market outlook [1], it is predicted that the total amount of fleet will be doubled in 2035 and the long-term demand will continue with a world average Revenue Passenger Kilometer (RPK) growth rate of 4.7 percent per year over 20 years. Without exception, Japanese air traffic in 10 years is also anticipated to be increased by 1.5 times of that in 2005 [2]. Increase of fuel consumption and greenhouse gas emission and safety deterioration in the congested airspace are major concerns which would be caused by the air traffic explosion. An innovative change is strongly demanded on the current air traffic system. In the world, various research and developments have been conducted under the NextGEN [3] and SESAR [4] programs in the United States and European countries, respectively. In Japan, Collaborative Actions for Renovation of Air Traffic System (CARATS) has been drawn up by the government as a roadmap towards the Japanese ideal Air Traffic Management (ATM). Trajectory Based Operation (TBO)

in which whole flight phase from departure to arrival is dealt with one continuous trajectory in the unified airspace is one of the remarkable potential to improve overall operational efficiency. Not only economic efficiency improvement but capacity enhancement is expected because all aircraft can fully achieve their performance by flying on the optimal trajectories including optimal flight route, altitude and speed profiles. In recent years, among Japanese researchers, arrival management system is focused on to support the TBO concept, and its effectiveness is analyzed considering actual situation with the real flight track data. The previous research indicate that operational efficiency of multiple aircraft can be improved while maintaining the requirement defined by Air Traffic Control Standard by assigning the appropriate arrival time on each aircraft [5]. Whether the arrival time control is possible or not is dependent on how accurately replicated the command of Flight Management System (FMS) can be on the ground side [6]. In other words, if the arrival time of multiple aircraft could be predicted precisely and the optimal arrival time calculated by considering the future situation around terminal would be assigned on those aircraft, overall operational efficiency could be improved since the arrival aircraft would not have to fly in inefficient manners arising from such the current manual adjustment of arrival time and sequencing. A function called Required Time of Arrival (RTA) [7] already equipped in some latest FMS suggests a possibility of the arrival time designation in real time. Rigorous aircraft performance model and calculation logic of FMS itself are required to replicate the command of FMS such as altitude and operational speed independently on the ground. These are highly-confidential and not disclosed; however, it is assumed that the flight trajectory based on the FMS command can be reproduced, i.e., trajectory prediction which is necessary for the precise arrival time prediction is possible to some extent by using trajectory optimization and performance model often-used in the ATM research. In the next step for the optimal arrival time assignment, appropriate arrival time must be calculated in light of overall efficiency such as total operational cost of multiple aircraft. The trajectory prediction tool developed by the author's research group is capable of giving fuel-optimal trajectories with various terminal time by changing a weighting parameter. This parameter is relevant to the Cost Index (CI) [8] set in the FMS. The result such that the predicted trajectory fits well with the actual one in the certain range of CI value has been already obtained; though, the coincidence in the wider range available has not been investigated yet. Therefore, this research aims to demonstrate a methodology to estimate CI values of actual flights by a developed trajectory prediction tool by confirming the coincidence of estimated CI values for multiple aircraft types.

## 2. DATA FOR FLIGHT ANALYSIS

### 2.1 ADS-B flight track data

This research uses actual flight track data broadcasted from airborne aircraft through Automatic Dependent Surveillance-Broadcast (ADS-B) system. The system is expected to be used as next-generation surveillance system since ADS-B broadcasts accurate position and time information measured via Global Positioning System (GPS). Although ADS-B flight data can be obtained easily by a general antenna used for receiving digital terrestrial broadcast, observable range is limited by the antenna's performance. Hence, this research uses the ADS-B data uploaded on the web site, Flightradar24.com [9]. As it is well known these days, this web service provides the ADS-B equipped aircraft's information recorded by the cooperators' and the company's receivers installed all over the world. The data provided by Federal Aviation Administration (FAA) and Multi LATeration (MLAT) data are partially available. The information given as the data file is time, callsign, departure and arrival airport name, GPS position, pressure altitude, ground speed and track angle. The airdata such as True AirSpeed (TAS), Calibrated AirSpeed (CAS) or Mach number are not recorded in the data file; though, they can be estimated by applying meteorological data to the recorded ground speed data.

### 2.2 Analysis object

This paper focuses on the flights on the aRea NAVigation (RNAV) route Y20 which is one of the Japanese domestic major enroutes connecting Tokyo International Airport with Fukuoka International Airport. Figure 1 shows a part of Y20 enroute. 39 flight cases between two waypoints of Y20, SEKID and STOUT flying without any altitude and route change are extracted for the analysis because the trajectory prediction tool presupposes to replicate the trajectory generated by FMS.

## 3. TRAJECTORY PREDICTION

The developed trajectory prediction tool consists of the two methods, flight parameters estimation and flight trajectory optimization. The actual flight time can be directly obtained from the recorded data; on the other hand, the operationally-important airdata such as CAS and Mach number must be estimated with acceptable accuracy. Those airdata can be estimated from time and position data or the recorded ground speed data with certain meteorological data and aircraft performance model in the flight parameters estimation.



Figure 1 RNAV enroute Y20 and the waypoints

In the next step, flight trajectory optimization is implemented using identical meteorological data and performance model. Objective function for the optimization is set to be a combination of fuel mass consumed and weighted flight time. The weighting parameter which determines a trade-off between fuel and time corresponds to the Cost Index used in the actual operation. Changing this parameter gives fuel-optimal trajectories with various flight time as well as the trade-off between fuel and time can be adjusted by the different CI values. The fuel-optimal trajectory which has the closest CAS trajectory to the actual one is selected in the trajectory prediction process. If the error of CAS trajectory is small enough, it is proved that CI value of the actual flight has been estimated. Prediction accuracy of arrival time can be calculated as well. This trajectory prediction method which consists of flight parameters estimation and trajectory optimization is explained in Figure 2.

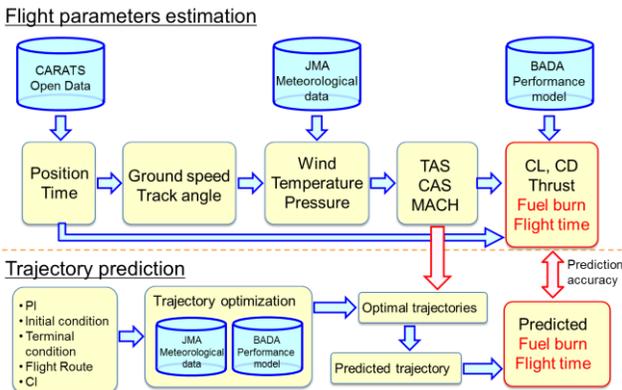


Figure 2 Relationship between feasible solutions and weighting parameter

### 3.1 Flight parameters estimation

For the purpose of accurate arrival time prediction, airspeed information such as CAS or Mach number should be estimated as precisely as possible. Although ground

speed can be calculated by differentiating the position data, this analysis uses the ground speed data stored in the ADS-B data file. Then, airspeed data represented by TAS, CAS and Mach number can be estimated by applying meteorological wind velocity and temperature data to the ground speed data. Furthermore, performance calculation is carried out with an aircraft performance model to obtain drag, thrust force and fuel flow. Numerical Prediction Grid Point Value (GPV) Meteorological Data released from the Japan Meteorological Agency (JMA) [10], and Base of Aircraft Data (BADA) model [11] developed and maintained by the European Organization for the Safety of Air Navigation are used as the meteorological data and aircraft performance model, respectively. Accuracy of the estimated flight parameters are directly influenced by the quality of meteorological GPV data and BADA model. According to the previous three investigations [12-14], it has been revealed that those model and data have satisfactory quality to obtain the flight parameters accurate enough to be used in the flight analysis.

This flight parameters estimation is applied to the objective flights to estimate the actual trajectory. The estimated parameters inevitably include errors arising from the meteorological data, performance model and initial mass difference. Among those factors, influence by the third factor has not been confirmed yet. Since the actual initial mass is unknown, 80% of reference mass listed in the BADA model is assumed at the pressure altitude point of 10,000 [ft].

### 3.2 Flight trajectory optimization

Although the calculation logic which gives the flight profiles is unknown due to confidentiality of FMS, the flight trajectory calculation is performed on the basis of a certain evaluation index. Hence, this research assumes that the trajectory of FMS can be replicated by flight trajectory

optimization to some extent. This section describes the flight trajectory optimization method which is a part of trajectory prediction proposed in this paper.

Firstly, the performance index which should be minimized is defined as combination of fuel consumption and flight time.

$$J = \int_{t_0}^{t_f} [\mu(t) + a] dt \quad (1)$$

Here,  $\mu[\text{kg/s}]$  is fuel flow and  $a$  is a weighting parameter used to adjust flight time. Trajectory optimization calculation finds the fuel minimum trajectory from feasible trajectories for given value of the parameter  $a$ , consequently, the optimal trajectories which have various flight time can be obtained as a trade-off between fuel burn and time. This weighting parameter is strongly related with the Cost Index. The performance index is also described as the cost of flight,

$$J_{\text{dollars}} = \int_{t_0}^{t_f} \left[ \frac{1}{100} \frac{C_{\text{fuel}}}{0.4536} \mu(t) + \frac{C_{\text{time}}}{0.4536} \right] dt \quad (2)$$

where  $C_{\text{fuel}}$  is the fuel cost in [cent/lb] and  $C_{\text{time}}$  is the time cost in [dollars/hour]. Since CI is defined as time cost per fuel cost, following relationship is established between  $a$  and CI by comparing the Eqs. (1) and (2).

$$CI = \frac{C_{\text{time}}[\text{dollars}/\text{hour}]}{C_{\text{fuel}}[\text{cent}/\text{lb}]} = 79.37a \quad (3)$$

Figure 3 explains the trade-off between fuel and time. CI or the weighting parameter is equivalent to the slop of tangent of the feasible solutions' boundary, i.e., the optimal

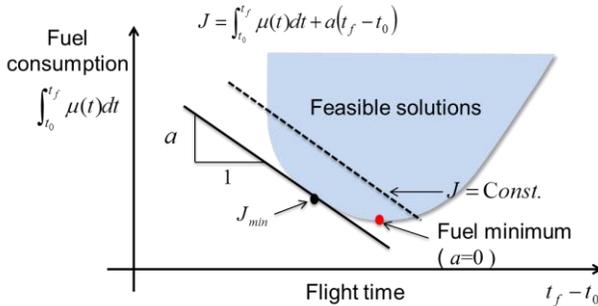


Figure 3 Relationship between feasible solutions and parameter  $a$  solutions for various weighting parameters form a Pareto front. In this manner, cost index is a free parameter for each performance index to be optimized.

Aircraft motion is described by three degree of freedom (3DOF) governing equations defined with point mass approximation. State variables are CAS and pressure altitude, and thrust and flight path climb angle are taken as the corresponding control inputs because the object aircraft is assumed to fly on the pre-defined enroute. The governing equations are denoted in Eqs. (4) to (7).

$$\frac{dx}{dt} = V_{TAS} \cos \gamma \quad (4)$$

$$\frac{dH}{dt} = V_{TAS} \sin \gamma \quad (5)$$

$$m \frac{dV}{dt} = T - D - mg \sin \gamma_a - m \frac{dV_w}{dt} \cos \gamma_a \quad (6)$$

$$m \frac{d\gamma}{dt} = L - mg \cos \gamma_a = 0 \quad (7)$$

$V_{TAS}$  is transformed from CAS given as the state variable. Eq. (7) describes quasi-steady assumption for the path angle.

Various optimization methods exist in the field of optimal control theory. This research uses Dynamic Programming (DP) as an easy-to-handle optimization method. The optimal trajectory is determined by selecting state variable grid points generated in a state space such that the performance index Eq. (1) is minimized. Although DP guarantees global optimality if the calculation is implemented for every feasible grid point combination, computational load explodes with increasing number of state variables. To avoid this computational difficulty which is known as the curse of dimensionality, the Moving Search Space DP (MS-DP) method invented in the previous research [15] is applied.

### 3.3 Trajectory prediction

The fuel optimal trajectories with various flight time are obtained by the DP trajectory optimization calculation. Now, the most probable trajectory must be selected from those trajectories to replicate the actual trajectory generated by FMS. This step is called trajectory prediction in this paper. Difference between optimal and actual CAS trajectories which is one of the operationally-important airdata is used to find the closest optimal trajectory to the actual trajectory. Figure 2 in the section 3.1 explains the proposed trajectory prediction method with the flight parameters estimation and trajectory optimization.

## 4. COST INDEX ESTIMATION

### 4.1 Calculation condition

Calculation condition in the trajectory optimization is listed in Table 1.

The purpose of this analysis is to replicate the FMS command; therefore, no lateral deviation from pre-defined enroute is considered, and the waypoints where altitude and velocity are optimized are set on the enroute Y20 at approximately 20[km] intervals.  $H_{\text{cruise\_act}}$  indicates the actual cruise altitude, and  $V_{\text{est\_SEKID}}$  and  $V_{\text{est\_STOUT}}$  are the initial and final values of CAS obtained by the flight parameters estimation.  $m_{\text{SEKID}}$  is the aircraft mass at SEKID. 80% of BADA reference mass is assumed at the altitude of 10,000 [ft] point which is already passed before SEKID.

Table 1 Calculation condition

Route		Y20 (SEKID to STOUT)	
Flight distance	$X_0$ :	0	$X_f$ : 789.1 [km]
	$\Delta X$ :	approx. 20 [km]	
Altitude	$H_{min}$ :	-	$H_{max}$ : $H_{cruise\_act}$
	$\Delta H$ :	400 [ft]	
CAS	$V_{min}$ :	BADA	$V_{max}$ : $V_{est\_max}$
	$\Delta V$ :	2 [kt]	
Initial	$H_0$ :	$H_{SEKID}$	$V_0$ : $V_{est\_SEKID}$
	$m_0$ :	$m_{SEKID}$	
Final	$H_f$ :	$H_{STOUT}$	$V_f$ : $V_{est\_STOUT}$
	Weighting	$a$ : 0 to 2 (Step size: 0.01)	

4.2 Estimated Cost Index

The estimated CI values for four aircraft types are listed in Table 2. Figure 4 shows bar plot of the table. These 13 cases are extracted from the all 39 analysis cases by reason that trajectory prediction has been succeeded with small difference of CAS trajectory. From this result, it can be seen that the small category aircraft such as A320 and B738 takes lower CI values; on the other hand, the large category aircraft B772 selects comparatively higher values. Although accuracy of the estimated CI values cannot be calculated, these obtained values are mostly within the range of typical values introduced in the two aircraft companies' documents [8, 16].

Table 2 Estimated CI (13 cases)

	$30 \leq CI < 50$	$50 \leq CI < 70$	$70 \leq CI < 90$	$90 \leq CI < 110$
A320	32.8	52.0		
	44.8			
B738	36.8	60.0		
	42.4			
B763		66.4	76.8	
			80.8	
B772			86.4	92.0
			87.2	104.0

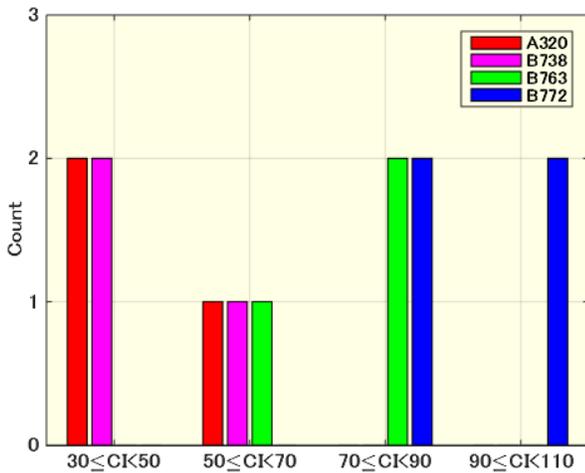


Figure 4 Histogram of estimated CI values for each aircraft type

Figures 5 to 8 indicate the comparison of actual and predicted trajectories of pressure altitude, CAS and Mach number in each aircraft type. These are the represented cases extracted from the successful 13 cases. In each case, actual trajectories are well replicated by the trajectory prediction with good coincidence of CAS trajectory. From Figures 5 and 6, the selected CI values and cruise Mach number are not much different; though, each cruise altitude differs by 2000[ft]. The selected CAS of A320 at cruise phase is higher than that of B738 by approximately 20[kt] and descent CAS values are totally different. This explains that the selected speed and altitude are different in aircraft types even with similar CI values. In Figures 7 and 8, the typical operation with higher CI can be seen in the constant part of CAS and Mach number. The transition part between constant CAS and Mach number is not replicated perfectly; nevertheless, approximate tendency is represented by the predicted trajectory. The predicted CAS at climb phase mostly coincides with the actual value of about 310[kt] and 300[kt] in each case; on the other hand, the predicted CAS trajectory gradually increases in the descent phase while the actual CAS takes constant value. In the cruise phase, depending on the different value of maximum operating Mach number (MMO), the predicted trajectory of Mach number follows the actual trajectory in each case. Since predicted altitude agrees completely, cruise CAS trajectories are appropriately predicted in both cases.

5. CONCLUSION

This research demonstrated a proposed methodology to estimate CI values of actual flights by a developed trajectory prediction tool with ADS-B flight data. Since the developed tool aims to independently replicate the FMS trajectory, analysis object has been limited in the cases which fly on the pre-defined route without any cruise altitude change. Among the corresponding 39 cases picked from the flights in August and September 2017, 13 flights can be admitted that the trajectory prediction has been succeeded with small difference of CAS trajectory. Cost Index values of those flights are estimated from the weighting parameter  $a$  set in the performance index of the tool. The actual CI value is not available; however, the estimated CI values are considered to be reasonable judging from the reference values of each aircraft type described in the aircraft companies' documents. From this analysis results, it has been revealed that the actual flight which obeys the FMS command can be replicated by the developed trajectory prediction tool, and the actual CI value can be estimated with a certain degree of accuracy as well. To enhance reliability and to improve versatility of the proposed CI estimation method, comparison with the actual CI values are indispensable. At the same time, the prediction tool must be improved in terms of the factors

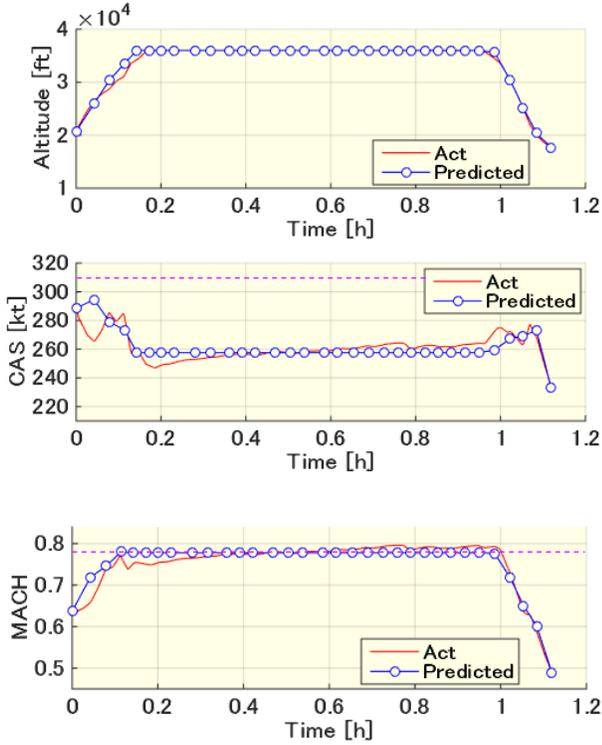


Figure 5 Actual and predicted altitude, CAS and Mach (A320, CI32.8)

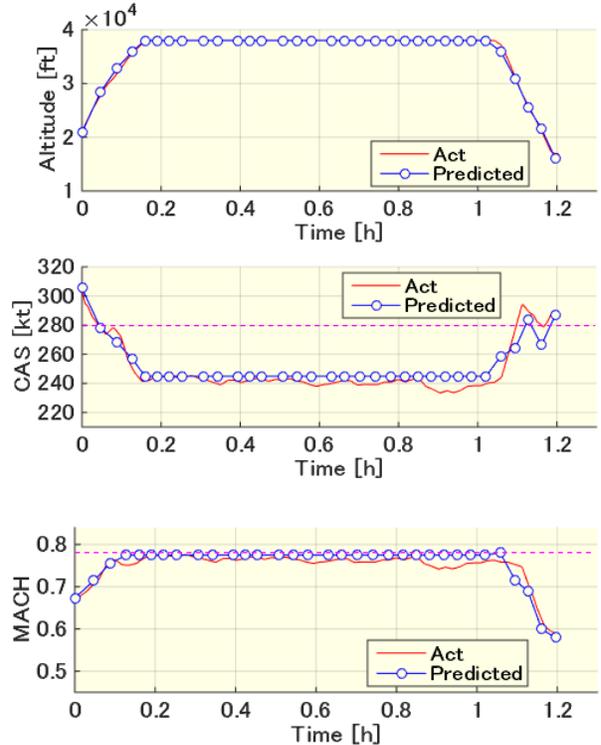


Figure 6 Actual and predicted altitude, CAS and Mach (B738, CI36.8)

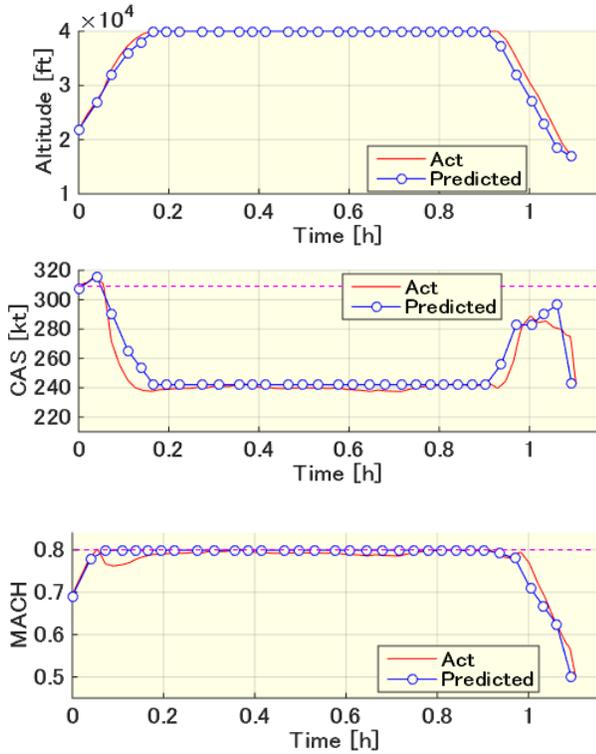


Figure 7 Actual and predicted altitude, CAS and Mach (B763, CI80.8)

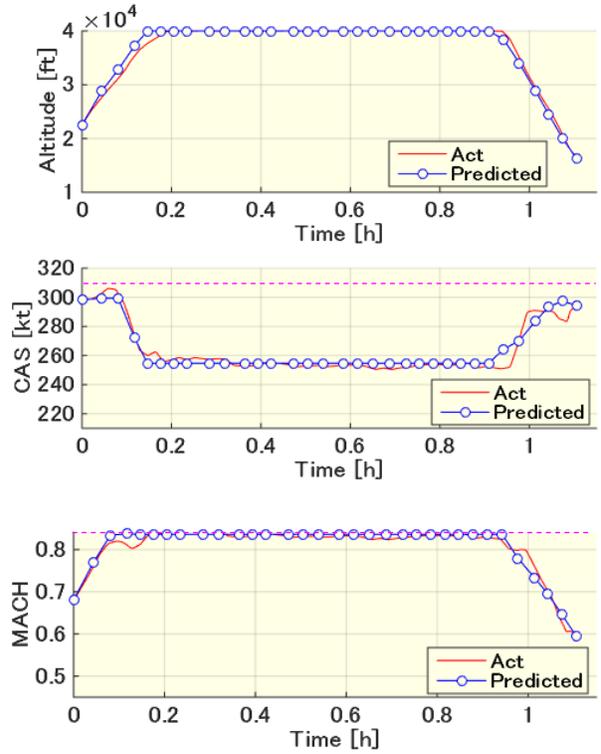


Figure 8 Actual and predicted altitude, CAS and Mach (B772, CI86.4)

which determine quality of the tool such as accuracy of aircraft performance model and meteorological data and the calculation logic itself. These future works still remain; though, this paper is of significance in that the actual CI values can be estimated from the given radar track data by the optimization based trajectory prediction method. The estimated CI values are extremely useful for quantification of potential benefits which will be gained in the more efficient air traffic system.

## 6. ACKNOWLEDGMENTS

This research used Numerical Weather Prediction GPV Data released by the Japan Meteorological Agency and BADA model developed by EUROCONTROL to reconstruct flight parameters from CARATS Open Data. These organizations' support to the research is greatly appreciated.

## 7. REFERENCES

- [1] The Boeing Company, "Current Market Outlook 2017-2036", URL: [http://www.boeing.com/resources/boeingdotcom/commercial/market/current-market-outlook-2017/assets/downloads/2017-cmo-compressed\\_091917.pdf](http://www.boeing.com/resources/boeingdotcom/commercial/market/current-market-outlook-2017/assets/downloads/2017-cmo-compressed_091917.pdf).
- [2] Study Group for the Future Air Traffic Systems, Long Term Vision for the Future Air Traffic Systems, CARATS, Collaborative Actions for Renovation of Air Traffic Systems, URL: <http://www.mlit.go.jp/common/000123890.pdf>, 2010.
- [3] Federal Aviation Administration, Next Generation Air Transportation System (NextGEN), URL: <http://www.faa.gov/nextgen/>
- [4] The Single European Sky ATM Research (SESAR), URL: <http://www.sesarju.eu/>
- [5] Miyazawa, Y., Matsuda, H., Shigetomi, S., Harada, A., Kozuka, T., Wickramasinghe, N.K., Brown, M. and Fukuda, Y., "Potential Benefits of Arrival Time Assignment -Dynamic Programming Trajectory Optimization applied to the Tokyo International Airport-", 11th USA/Europe Air Traffic Management Research and Development Seminar (ATM Seminar 2015), Lisbon, 2015.
- [6] Higuchi, Y., Tamura, K., Kitazume, N., Miyazawa, Y. and Brown, M., "A Study on Arrival Time Control with FMS for Efficient Arrival Management", in Japanese, JSASS 54th Aircraft Symposium, Toyama, 2016.
- [7] Jackson, Michael R. C., "Airborne Required Time of Arrival (RTA) Control and Integration with ATM",

- 7th AIAA Aviation, Technology, Integration, and Operations (ATIO) Conference, Belfast, September, 2007.
- [8] The Boeing Company, Roberson, B., "Fuel Conservation Strategies: Cost Index Explained", Aero Quarterly, QTR 02-07, 2007.
- [9] Flightradar24, URL: <http://www.flightradar24.com/>
- [10] Japan Meteorological Business Support Center Online Data Service, URL: <http://www.jmbssc.or.jp/en/index-e.html>
- [11] EUROCONTROL Experimental Center: User Manual for the Base of Aircraft Data (BADA) Revision 3.11, EEC Technical/Scientific Report, No.13/04/16-01, 2013.
- [12] Totoki, H., Kozuka, T., Miyazawa, Y. and Funabiki, K., "Comparison of JMA Numerical Prediction GPV Meteorological Data and Airliner Flight Data", in Japanese, JSASS Kouku-Uchu Gijutsu, vol.12, 2013, pp. 57-63.
- [13] Harada, A., Miyamoto, Y., Miyazawa, Y. and Funabiki, K., "Accuracy Evaluation of an Aircraft Performance Model with Airliner Flight Data", Trans. JSASS Aerospace Technology Japan, vol.11, 2013, pp.79-85.
- [14] Tamura, K., Harada, A., Higuchi, Y., Matsuda, H. and Miyazawa, Y., "A study on aircraft performance model by using cargo flight Data", in Japanese, JSASS Kouku-Uchu Gijutsu, vol.16, 2017, pp. 27-36.
- [15] Miyazawa, Y., Harada, A., Wickramasinghe, N.K., Miyamoto, Y., "Effect of Jet Passenger Aircraft Performance Model on the Optimal Periodic Cruise Maneuver", in Japanese, JSASS Kouku-Uchu Gijutsu, vol.12, 2013, pp. 99-105.
- [16] Airbus Flight Operations Support & Line Assistance, Customer Services Directorate, "Getting to grips with the cost index", issue II, May 1998.

## 8. COPYRIGHT

### Copyright Statement

The authors confirm that they, and/or their company or institution, hold copyright of all original material included in their paper. They also confirm they have obtained permission, from the copyright holder of any third party material included in their paper, to publish it as part of their paper. The authors grant full permission for the publication and distribution of their paper as part of the EIWAC2017 proceedings or as individual off-prints from the proceedings.