# [EN-A-057] Prediction of Flight Time Uncertainty for 4D Trajectory Management

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**Abstract:** The feasibility of flight time uncertainty prediction based on flight and weather conditions is discussed. Level flight trajectories with near constant Mach number and flight direction are extracted from the actual track data. The flight time error are analyzed as the difference between the inferred intent flight time and the actual flight time. Through cluster analysis, data sets representing similar flight and weather conditions and corresponding flight time uncertainty are obtained. It is demonstrated using these data clusters that the accurate prediction of the flight time uncertainty becomes possible if the flight and weather conditions are correctly applied. An artificial neural network is applied in order to develop a regression model between the flight and weather conditions and the flight time uncertainty. This regression model is developed intentionally to predict the flight time uncertainty over a short distance in order to enable a prediction over a long distance by its integration. Through validation analysis, it is clearly demonstrated that the presented model is able to predict the flight time uncertainty without overestimation or underestimation even in calm or sever conditions, where the conventional model inevitably overestimates or underestimates to result in potential loss of efficiency or safety.

**Keywords:** air traffic management, 4D trajectory, flight time uncertainty, prediction, neural network

## 1. INTRODUCTION

In the long-term visions for the future air traffic management systems [1-3], it is planned to introduce the time-based operation in order to cope with the increase of the air transportation demand. The 4D trajectory management is expected as one of the promising methodologies for the time-based operation. Although the flight time is managed in the 4D management, some uncertainty in the flight time inevitably arises [4] due to the stochastic influence, e.g. departure time uncertainty, fluctuations of weather condition, etc.

In this paper, the flight time uncertainty is especially focused among various types of uncertainty. In addition, the feasibility to improve the safety and efficiency by introducing a condition-based model, which denotes the model to appropriately predict the flight time uncertainty according to flight and weather conditions in this paper, is investigated. Such the condition-based model was referred to improve both the safety and efficiency between an aircraft pair in the free-flight [5]. In past studies, the standard deviation of the flight time was treated as the evaluation index of its uncertainty, and it was theoretically proven that the standard deviation increases in proportional to the flight distance or time [4, 6]. Empirical data analyses have also validated the almost linear increase of the flight time uncertainty [7-9]. However, the models of the flight time uncertainty are treated assuming the constant conditions in these studies. Concerning the condition-based modeling, some studies have attempted modeling of the wind uncertainty from the ensemble weather forecast data [10,11]. In contrast to these studies, the authors recently presented the feasibility to predict the flight time uncertainty for a certain distance directly from the weather condition data such as tailwind, crosswind, temperature and pressure [12]. In this study, the feasibility to predict the flight time uncertainty for a long distance as the summation of the uncertainty prediction for short distances. The flight time uncertainty in the ground-based trajectory prediction is especially focused. Firstly, the flight time uncertainty at the distance of 100km in evaluated, and the advantage of the condition-based modeling is evaluated. Then a regression model of the flight time uncertainty at the distance of 10km is developed, and the feasibility to predict the flight time uncertainty at the long distance of 100km by the summation of the prediction at the short distance of 10km. Actual track data and weather forecast data are applied in analyses. The cluster analysis and the artificial neural network are employed for regression. Finally, the effectiveness of the developed model is verified using trajectory data.

## 2. DATA PROCESSING

### 2.1 Data for Analysis

For the analyses in this study, the flight data collected by the SSR Mode S [13], placed in Tokyo and Sendai in Japan, in March, June, September and December in 2015 are applied. This data include the aircraft type, indicated air speed (IAS), true air speed (TAS), ground speed (GS), Mach number, pressure altitude, azimuth angle, true track angle etc. recorded at every ten seconds. For the weather forecast data, the Global Spectral Model (GSM) of the numerical forecast data [14] are applied. The GSM data are updated every 6 hours providing the temperature and wind information every 3 hours at grid points placed every 0.25deg in longitude and 0.2deg in latitude at every 50~100 hPa pressure altitudes. In the analyses, the forecast values are calculated by linear interpolation on time, longitude, latitude and pressure altitude.

### 2.2 Flight Time Error Analysis

The flight time error Δt is defined as the difference between the intent flight time $t_{int}$ and the actual one $t_{act}$ as (1).

$$\Delta t \triangleq T_{act} - T_{int} \tag{1}$$

Assuming the GS data in the SSR Mode S are correct, the actual flight time $t_{act}$ is calculated through the time integration of the recorded actual GS $GS_{act}$ as (2).

$$\int_0^{T_{act}} GS_{act} dt = D \tag{2}$$

On the other hand, no information on flight intent are included in the trajectory data. Thus, it is necessary to estimate the intent GS to calculate the intent flight time $t_{int}$ for each trajectory data. In order to estimate the intent flight time, the flight trajectories continuously satisfying the following conditions for more than 100km are extracted regarding them holding a flight intent: pressure altitude above 25000ft, true track angle in 30~150deg, maintaining IAS within 5kt, true track angle within 1.5deg and pressure altitude within 100ft. Consequently, 33791 trajectories shown in Fig. 1 are extracted.

To calculate the intent GS, in this paper, the intent IAS $IAS_{int}$ is calculated as the mean of IAS during the extracted trajectory. Then $IAS_{int}$ is translated into $TAS'_{int}$ using temperature and pressure [15]. The intent TAS $TAS_{int}$ is obtained through correction of $TAS'_{int}$ into flight direction by (3).

$$TAS_{int} = TAS'_{int} \cos\theta_W \tag{3}$$

where $\theta_W$ is the gap between the azimuth and track angle. The intent GS $GS_{int}$ speed is calculated as the sum of the intent TAS $TAS_{int}$ and the forecasted tailwind speed $W_{tl}$. The intent flight time $t_{int}$ at a certain flight distance $D$ is calculated through (4).

$$\int_0^{T_{int}} GS_{int} dt = D \tag{4}$$

As a result, the flight time errors of the extracted trajectories are obtained as their histogram shown in Fig.2. The mean and standard deviations and rms of the flight time error are -0.093sec, 6.63sec and 6.63sec, respectively. These show that the flight time error follows almost zero-mean and mound-shaped distribution. Because the actual data are processed throughout this study, the mean values can never become exactly zero even though they are quite small. Therefore, in this study, the rms value will be treated as the measure of the flight time uncertainty.

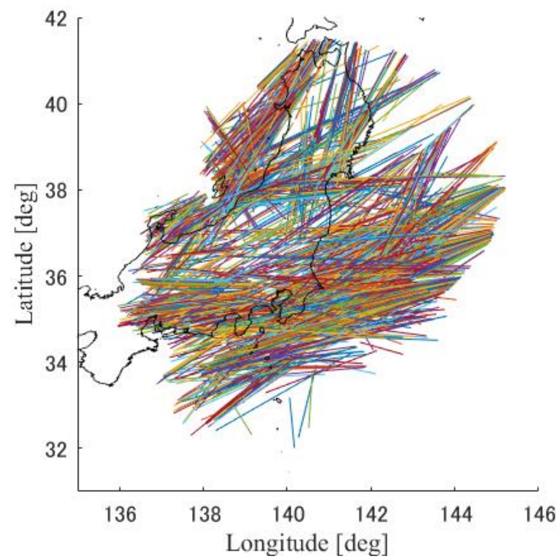


Figure 1 Extracted Trajectories

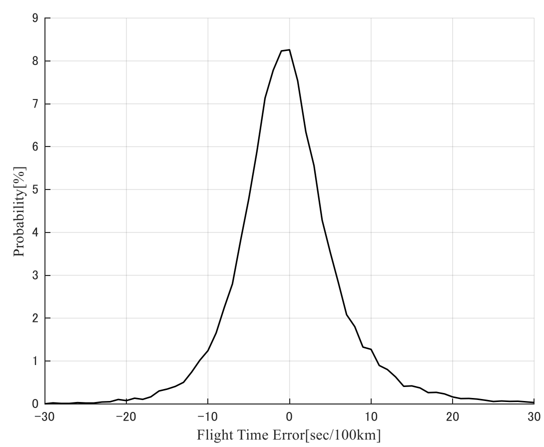


Figure 2 Histogram of Flight Time Error at 100km (N=33791)

### 2.3 Cluster Analysis

In order to evaluate the flight time uncertainty, a sufficient numbers of data groups including sufficient numbers of trajectory data to calculate the rms values of the flight time error. For appropriate evaluation, it is preferable that the data groups consist of trajectories in similar flight and weather conditions. For this purpose a cluster analysis is applied for the extracted trajectory data. In this study, the cluster method with the Gaussian mixture models (GMM) [16] is applied to two-thirds of the trajectory data to construct data clusters for the modeling in next chapter. The rest one-third data are reserved for validation to be discussed in the chapter 4.

For the clustering parameters, the tailwind speed $W_{tl}$, the absolute value of the crosswind speed $|W_{cr}|$, the temperature $T$, the intended GS $GS_{int}$ and the flight time error $\Delta t$ are selected. The pressure altitude $ALT$ is excluded because it is concentrating every 2000ft and has strong correlation between other parameters. The weather parameters are provided in the numerical weather forecast, and $ALT$ is calculated as the mean of those in each extracted trajectory. For accurate estimation of statistics about the distribution of flight parameters in each data cluster, it is desirable that each parameter distribution forms near-Gaussian distribution without outliers. This is the reason to employ the GMM cluster method, which adopts the Gaussian mixture distribution as the base distribution of each data cluster. Through the clustering applying 200 initial clusters and 0.05 for the posterior probability threshold, 159 clusters including more than 30 trajectories are extracted for modeling in next chapter. The distributions of clustering parameters in one example cluster is shown in Fig3. It is obvious that all parameters except for the pressure altitude form near-Gaussian mound-shaped distribution without large outliers.

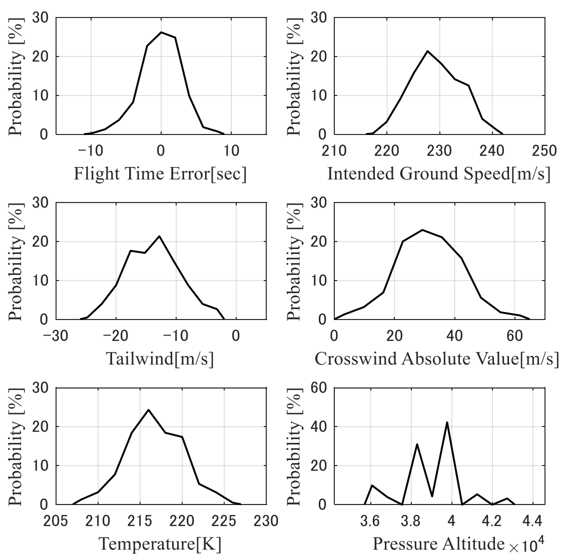



Figure 3 Histograms of Clustering Parameters of One Example Cluster (N=374)

## 3. FLIGHT TIME UNCERTAINTY MODELING

### 3.1 Theoretical Approach

The mathematical expression of the flight time is:

$$t = \frac{D}{TAS + W_{tl}} \tag{5}$$

Through the propagation of uncertainty law, the standard deviation of the flight time $\sigma_{\Delta}$ is obtained as :

$$\sigma_{\Delta t} = \frac{\sqrt{\sigma_{TAS}^2 + \sigma_{W_{tl}}^2}}{GS^2} D \tag{6}$$

This equation means that the flight time uncertainty is proportional to the flight distance and inversely proportional to the square of the GS. In the past studies, the factor of proportionality was treated as a constant value [4~7]. In the following sections, the effectiveness of the condition-based modeling of this factor and its feasibility are discussed.

### 3.2 Effectiveness Condition-Based Modeling

In order to evaluate the effectiveness of the condition-based modeling, the accuracy of the flight time uncertainty prediction using (a) constant value and (b) condition-based value are evaluated. In this study, coefficient of correlation $R$ and rms between the observed and predicted value are used as evaluation criteria of prediction accuracy. In the evaluation of the (a) case, the factor of proportionality was calculated as the value of $\sqrt{\sigma_{TAS}^2 + \sigma_{Wtl}^2}$ obtained using the whole trajectory data. This is obtained as 10.57. This modeling of the factor corresponds to that of the past studies [4~7]. In the evaluation of the (b) case, the values from each cluster is used for the factor of proportionality. This evaluation corresponds to the case that the flight and weather conditions were completely exactly calculated in the model.

The evaluation results assuming the distance of 10km of the two cases are shown in Fig. 4 as scatter plots of the observed and the predicted value. Their evaluation criteria are summarized in Table I. it is obvious that the prediction (b) is superior to the prediction (a). This result clearly validates the effectiveness of the introduction the flight and weather conditions into the prediction model.

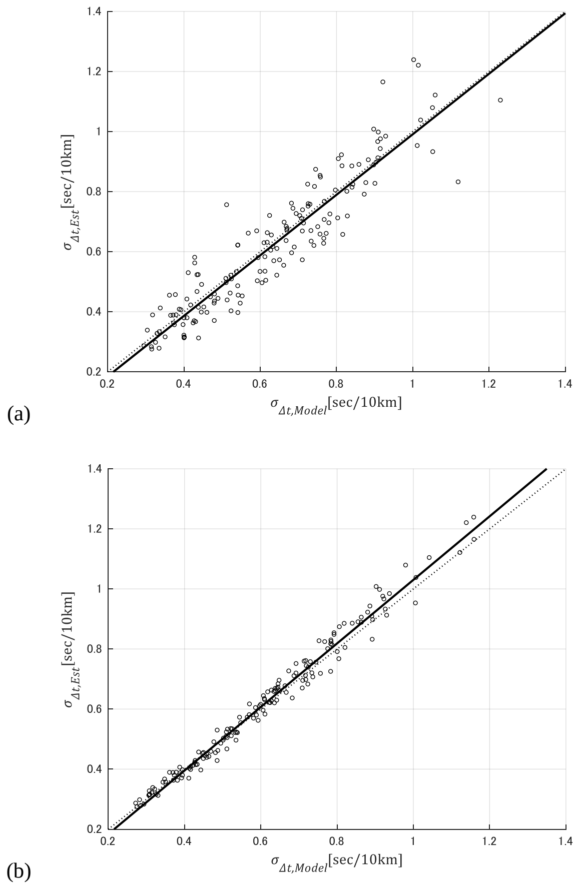


Figure 4 Regression Performance, a: Whole Trajectory Data, b: Each Cluster Data

Table I . prediction Performance

| | R | RMSE [sec/10km] |
|---|---|---|
| Whole Trajectory Data | 0.929 (p<0.000001) | 0.0822 |
| Each Cluster Data | 0.991 (p<0.000001) | 0.0324 |

### 3.3 Feasibility of Condition-Based Modeling

The feasibility of the condition-based modeling of the flight time uncertainty at the short distance of 10km is discussed in this section. In the derivation process of the uncertainty propagation model, the linearization of the flight time uncertainty and the independence among parameters are implicitly assumed. It is possible that these assumptions degrades the prediction accuracy inappropriately. Therefore, to avoid undesirable accuracy degradation, the artificial neural network (ANN) [16] is applied for regression analysis in this study because the ANN is able to approximate any nonlinear functions appropriately considering the correlation among parameters.

In this study, a single-hidden-layer ANN consisting of three layers of neurons is applied. Through the grid search, the number of neuron in the hidden layer was selected as 8. The hyperbolic tangent sigmoid transfer function is selected for the hidden layer, and a linear transfer function is applied for the output layer. For the ANN training, 10-fold cross-validation is applied. The regression result of the ANN between the condition-based modeling and the actual value is shown in Fig.5. The prediction performance is calculated as R = 0.940 (p<0.00001), RMSE = 0.0782 [sec/10km]. This result is better than those of the prediction by the constant value modeling, corresponding to the conventional prediction, and worse than the prediction using actual observed conditions, corresponding to the idealistic prediction discussed in the section 3.2. From this result, it is concluded that the prediction model using the ANN is able to predict the flight time uncertainty more accurately by using only the pre-flight information such as the weather forecast and flight plan. This clearly shows the feasibility of the condition-based prediction.

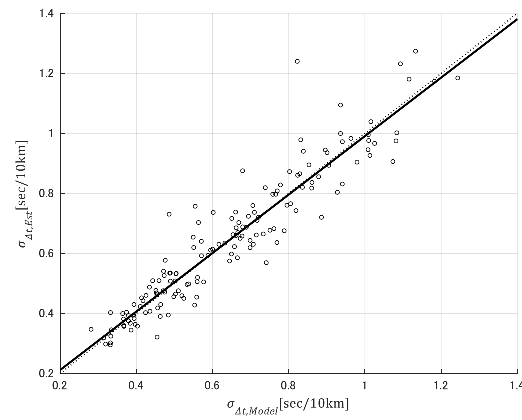


Figure 5 Regression Performance of ANN Model

## 4. ADVANTAGE of CONDITION-BASED PREDICTION

### 4.1 Validation of Condition-Based Modeling

In this chapter, the evaluations of the condition-based model is carried out to clarify its validity and effectiveness. For this evaluation, the reserved 11262 trajectories are applied. Firstly, for comparison, the constant value model is derived from the trajectory data for modeling, and the factor of the proportionality of rms for the distance of 100km has been obtained as 6.63[sec]. The prediction by the condition-based model is evaluated as the summation of the 10 ANN predictions for the distance of 10km.

For comparison of the predicted rms, the normalized flight time errors are introduced as the flight time error divided by the predicted rms for each trajectory. The histogram of the normalized flight time error is shown in Fig. 6. The histograms of the normalized flight time error are almost same. The rms values of normalized error of the constant value model and the condition-based model are 1.046 and 0.998 respectively. As the rms values for both cases are close to 1, both models are considered correct for the prediction of the overall flight time uncertainty.

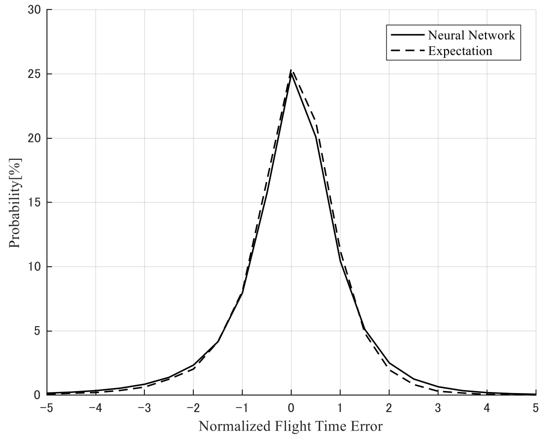



Figure 6 Histograms of Normalized Flight Time Error (N=11262)

## 4.2 Advantage of Condition-Based Prediction

To clearly evaluate the effectiveness of condition-based prediction, the trajectory data sets is divided into 5 layers at intervals of 20% in descending order of the predicted rms value of the condition-based prediction. The rms of the normalized flight time error in each layer are shown in Fig 7. It is obvious that the rms of the normalized flight time error using the constant value prediction increases with the layers. In the case that the rms value of the normalized flight time error is smaller than 1, a prediction is regarded overestimating the flight time uncertainty. This corresponds to the potential loss of the operational efficiency. In contrast, when the rms of the normalized error is larger than 1, a prediction is underestimating, and this means the potential loss of safety. The histograms of the normalized flight time error of both the largest and smallest 20% of the condition-based prediction cases are shown in Fig. 8. It is also found from these figures that the distributions of the constant value prediction becomes narrower/wider than those of the condition-based prediction in smallest/largest groups.

It is clearly demonstrated that the condition-based model is able to predict the flight time uncertainty correctly without large overestimation or underestimation whereas the constant value model inevitably overestimates or underestimates it.

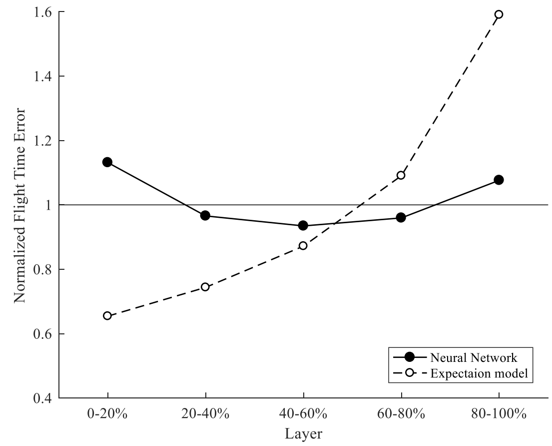
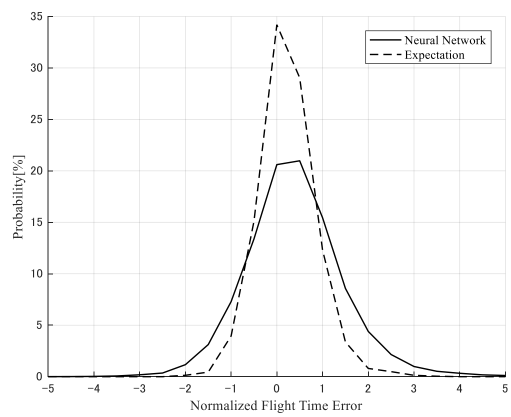
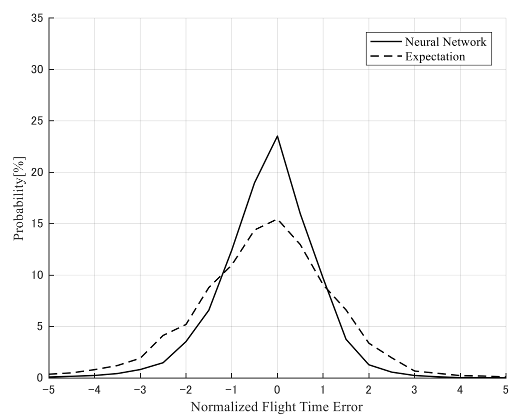



Figure 7 Each layer's rms of Normalized Flight Time Error, ANN model and Expectation model




(a)




(b)

Figure 8 Histograms of Normalized Flight Time Error at 100km, a: Smaller 20% Cases, b: Larger 20% Cases (N=429)

## 5. CONCLUSION

The feasibility to predict the flight time uncertainty based on the flight and weather conditions is clearly demonstrated through data analyses simulating the ground-based trajectory prediction. The cluster analysis using Gaussian mixture models enabled the accurate regression of the flight time uncertainty. It is validated that the summation of the flight time uncertainty for short distance trajectories is able to predict that for a longer distance trajectory, and that the presented condition-based prediction is able to evaluate the potential flight time errors accurately, while the conventional prediction using a constant proportionality factor inevitably overestimates or underestimates. It is also demonstrated that the presented condition-based prediction model is able to improve both safety and efficiency of the time-based operations. For future works, it is possible to improve the modeling performance by introducing or improving the prediction parameters. In addition, it is necessary to develop the prediction methodology to be capable of the ascent and descent trajectories.

As the numerical weather forecast is commonly available today, the presented prediction method is expected easy-to-materialize. The flight time uncertainty prediction and management are expected to improve various air traffic operations within future automated frameworks.

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