Probabilistic Conflict Detection in the Presence of Uncertainty

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Outline

- Background
- Purpose
- Probabilistic approach
- Simulation results
- Summary
- Future work

Background

- Current air traffic management system
  - ➢Human-operated system

- Increasing demand in air traffic
  - ➢Double to Triple in the next 25 years

→Workloads of air traffic controllers reach limits.

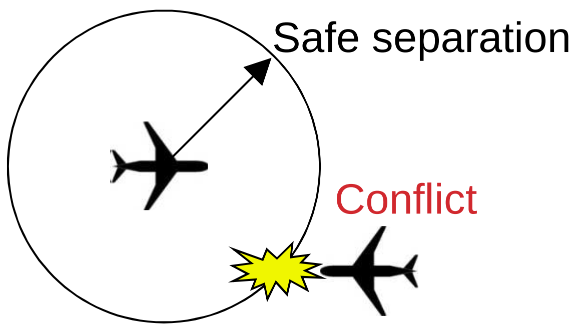
Background

- Automated air traffic control system
  - ➢Satisfy air traffic demands
  - ➢Alleviate the workloads of air traffic controllers

- Primary concern of air traffic control
  - ➢Monitor airspace for safety
  - ➢Maintain safe separation between aircraft

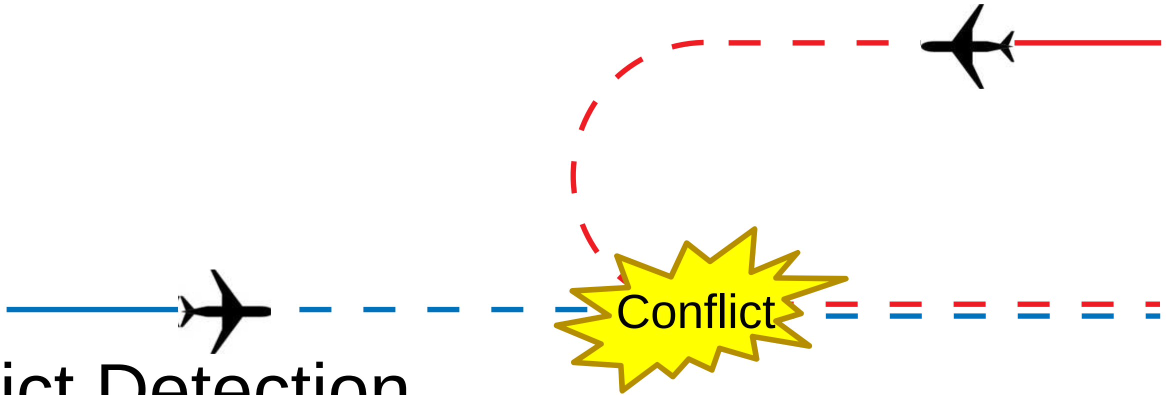
Avoid conflict between aircraft

→Automation of conflict detection



Purpose

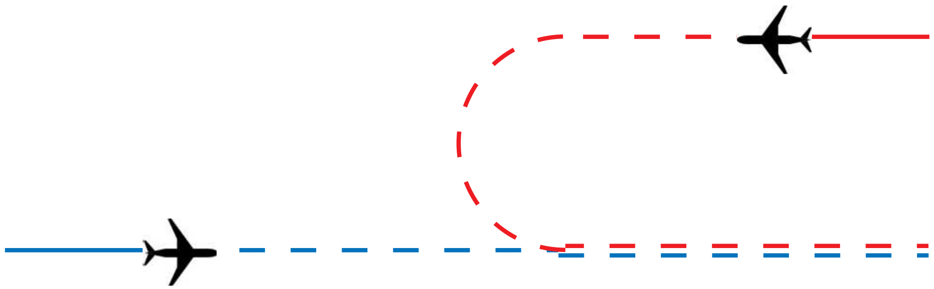
- Decision support tools
  - ➢Assist air traffic controllers
  - ➢Detect possible conflicts between aircraft



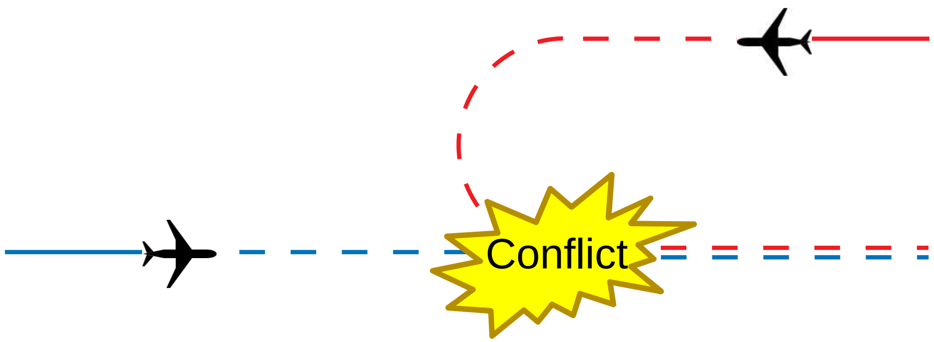
- Conflict Detection
  - ➢Merging (in Terminal Area)
  - ➢Detect potential conflicts
    by estimating aircraft's future trajectories

Purpose

1. Trajectory Prediction
   - Estimate aircraft's future trajectories

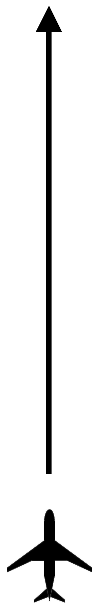


2. Conflict Detection
   - Detect possible conflicts

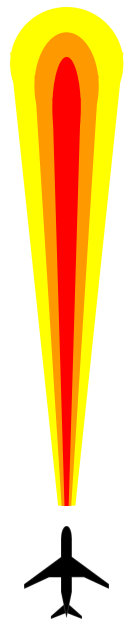


Trajectory Prediction

- Deterministic

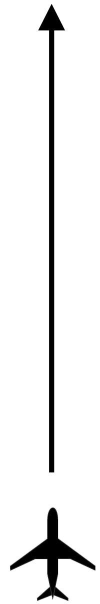


- Probabilistic



Trajectory Prediction

- Deterministic

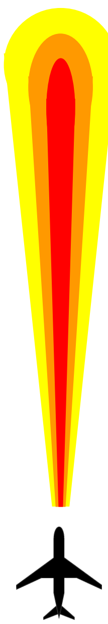


- ➢Fly along flight plans
- ➢Completely predict future positions of aircraft

→Optimistic predictions

Trajectory Prediction

- Probabilistic



➢ Consider various uncertainties
  - Wind
  - Measurement error
  - Navigation error
  - etc.

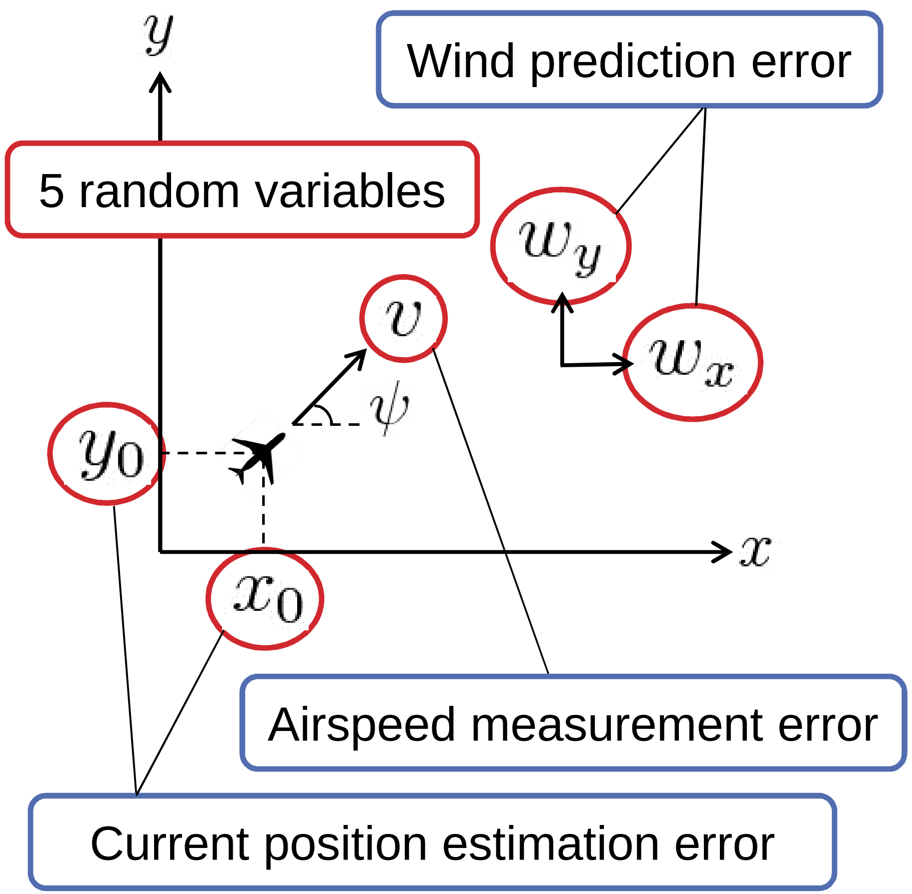
→More appropriate to model the aircraft's motion

Probabilistic Approach

- Uncertainties
  - Modeled as standard deviations of the Gaussian
  - Wind prediction error
    - 5.17 m/s
  - Airspeed measurement error
    - 5 knots (2.57 m/s)
  - Current position estimation error
    - 50 m laterally
  - Based on empirical air traffic data

Probabilistic Approach

- 2 dimensional horizontal plane



- Stochastic aircraft's motion (stochastic differential equation)

$$\dot{x} = v\cos\psi + w_x$$
$$\dot{y} = v\sin\psi + w_y$$

➢ Stochastic solution
  - Statistical information
    ➢ Expected value
    ➢ Variance
    ➢ Covariance

Probabilistic Approach

- Monte Carlo Simulation
  - Random sampling of random variables
  - Solve as deterministic problems on each sample point
  - Easy to implement, but high computational burden

- Generalized Polynomial Chaos
  - Preserve the ease of implementation
  - Reduce computational burden
    1. Polynomial approximation to determine the solution
    2. Collocation points as the sample points

Generalized Polynomial Chaos

- Polynomial approximation
  - Stochastic solutions are expressed as orthogonal polynomials of random variables.
  - M th order approximation

Stochastic solution : $z(p) = \sum_{m=1}^{M} C_m \Phi_m(p)$

Random variables (Uncertainties)

Coefficient

Polynomial basis function

Determined by collocation points

Generalized Polynomial Chaos

- Collocation points of random variables
  - ➢Not random sampling method (e.g. Monte Carlo Simulation)
  - ➢Strategically sampling method

Coefficient : $C_m = \sum_{i=1}^{Q} z(p_i)\Phi_m(p_i)w_i$

Total number of collocation points

Deterministic solution

Collocation point

Weights

  - ➢Collocation points and weights are determined uniquely based on extension of Gaussian quadrature

Generalized Polynomial Chaos

- Stochastic solution
  - Determined by Q deterministic solutions

$$z(p) = \sum_{m=1}^{M} C_m \Phi_m(p)$$

$$C_m = \sum_{i=1}^{Q} z(p_i)\Phi_m(p_i)w_i$$

  - Statistical information
    - Expected Value : $E(z) \approx C_1$
    - Variance : $V(z) \approx \sum_{m=2}^{M} {C_m}^2$

Generalized Polynomial Chaos

Generate collocation points and weights $p_i, w_i \ (i = 1, \cdots, Q)$

↓

Solve deterministic problems on each collocation point $z(p_i) \ (i = 1, \cdots, Q)$

↓

Calculate coefficient $C_m = \sum_{i=1}^{Q} z(p_i)\Phi_m(p_i)w_i$

↓

Generate approximate solution $z(p) = \sum_{m=1}^{M} C_m \Phi_m(p)$

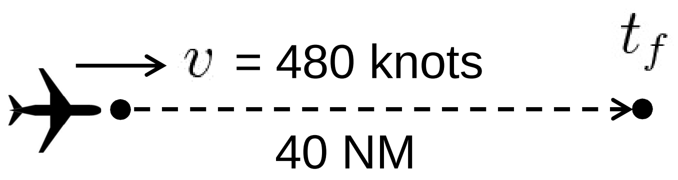
↓

Calculate statistical information of approximate solution

Expected value
Variance

Generalized Polynomial Chaos

- Example
  - ➢ Optimal control problem



5 random variables
- ➢ Wind prediction errors
- ➢ Airspeed measurement error
- ➢ Current position estimation errors

- Control input u : Bank angle
- Objective function : $J = 10^{-4} \cdot t_f + \int u^2 dt$

Bank angle

- Stochastic solution : t_f

Statistical information
- ➢ Expected value
- ➢ Standard deviation (Variance)

Generalized Polynomial Chaos

- Example
  - Generalized Polynomial Chaos

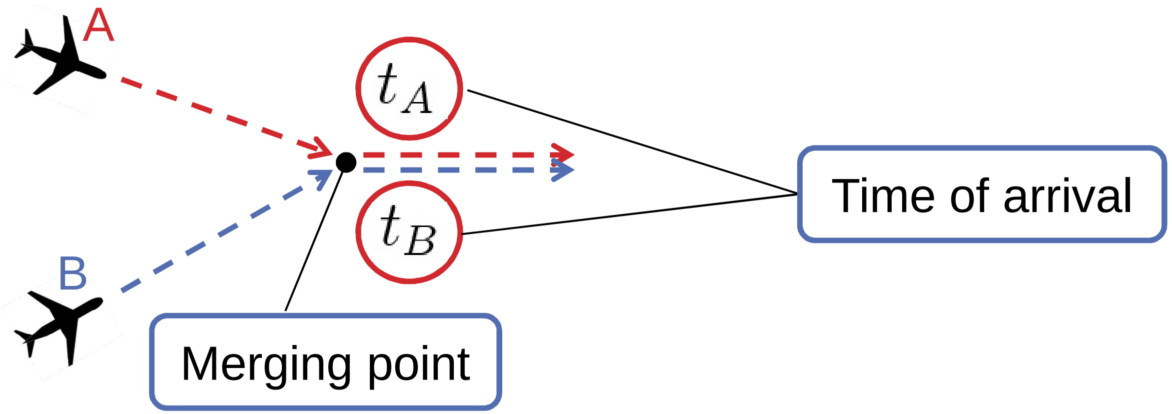
Total number of collocation points	Expected value	Standard deviation	Computation time
51	300.24 sec.	7.04 sec.	45 sec.
401	300.24 sec.	7.04 sec.	370 sec.

  - Monte Carlo Simulation

Total number of sample points	Expected value	Standard deviation	Computation time
25000	300.21 sec.	7.08 sec.	5.5 hr.

Trajectory Prediction

- Time of arrival at merging point



➢ Probability density of time of arrival

- Gaussian distribution

$$t_A : (t_{Amean}, \sigma_A)$$

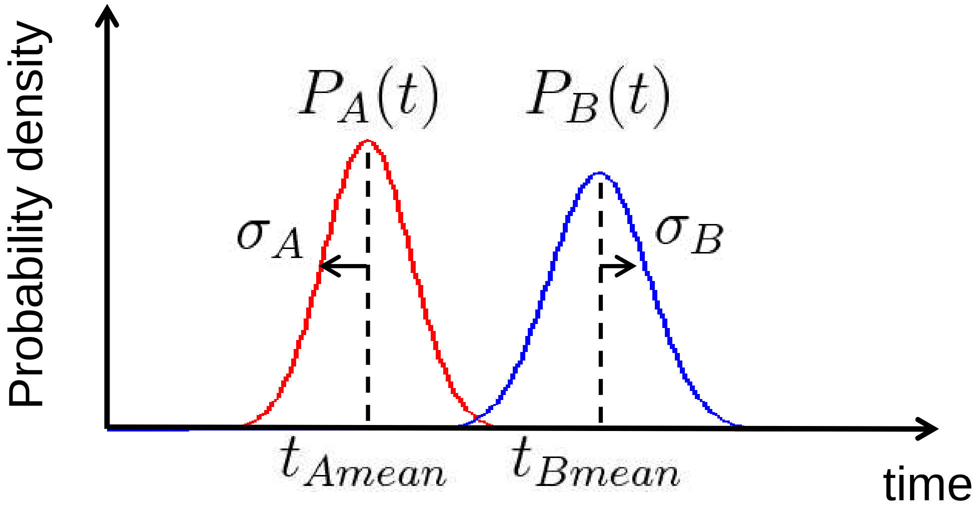
$$t_B : (t_{Bmean}, \sigma_B)$$

Expected value
(Mean of the Gaussian)

Standard deviation

Conflict Detection

- Conflict Probability at merging point



Safe separation (5 NM)

v = 600 knots

Time separation (τ = 30 sec.)

➢ Convolution integral

$$Conflict\ Probability = \int_{-\tau}^{\tau} P(T)dT$$

Time separation

$$P(T) = \int_{-\infty}^{\infty} P_A(t)P_B(t+T)dt$$

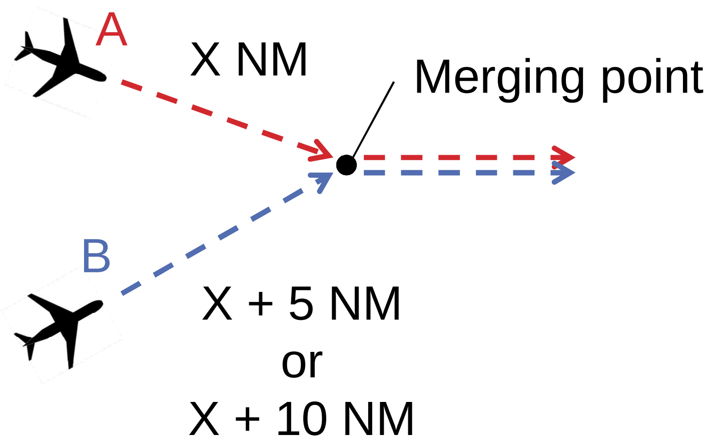
Simulation Results

- Merging situations
  - Case 1
    - Without meteorological predictions
  - Case 2
    - With meteorological predictions
      - Provided by the Japan Meteorological Agency
  - 51 collocation points for Generalized Polynomial Chaos
  - 5 random variables
    - Wind prediction errors
    - Airspeed measurement error
    - Current position estimation errors

Simulation Results

- Without meteorological predictions
  - Case 1

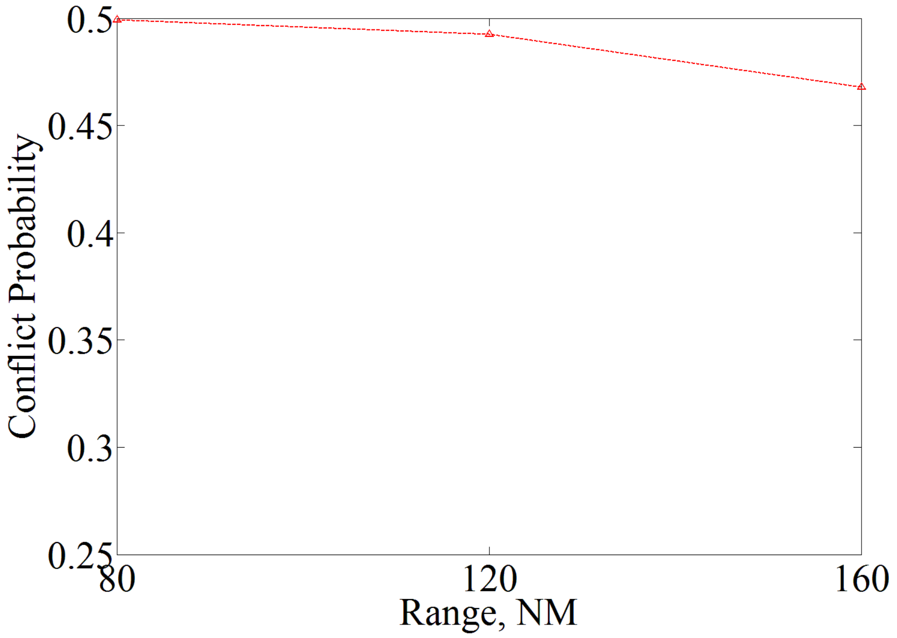
v = 480 knots



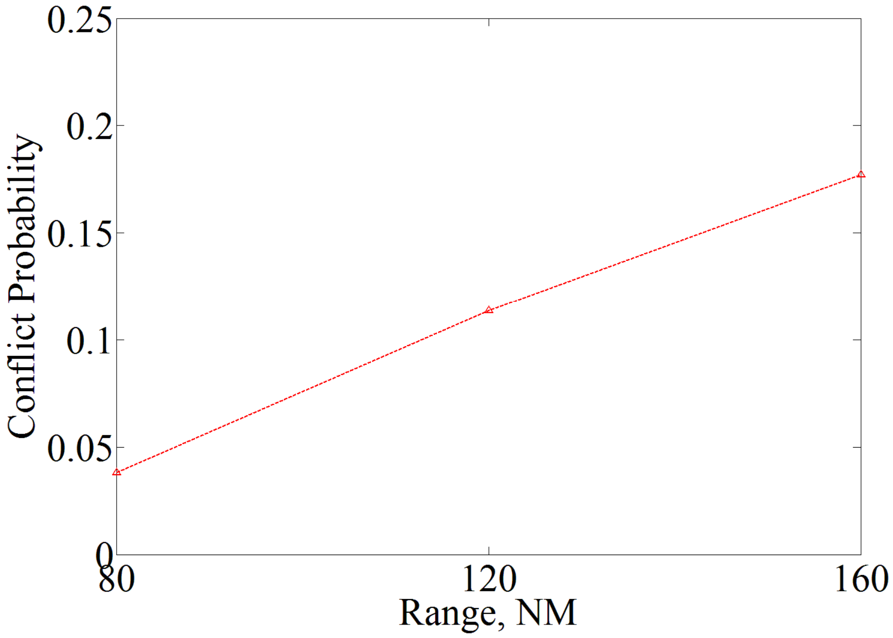
- X = 80, 120, 160 NM

Simulation Results

- 5 NM separation

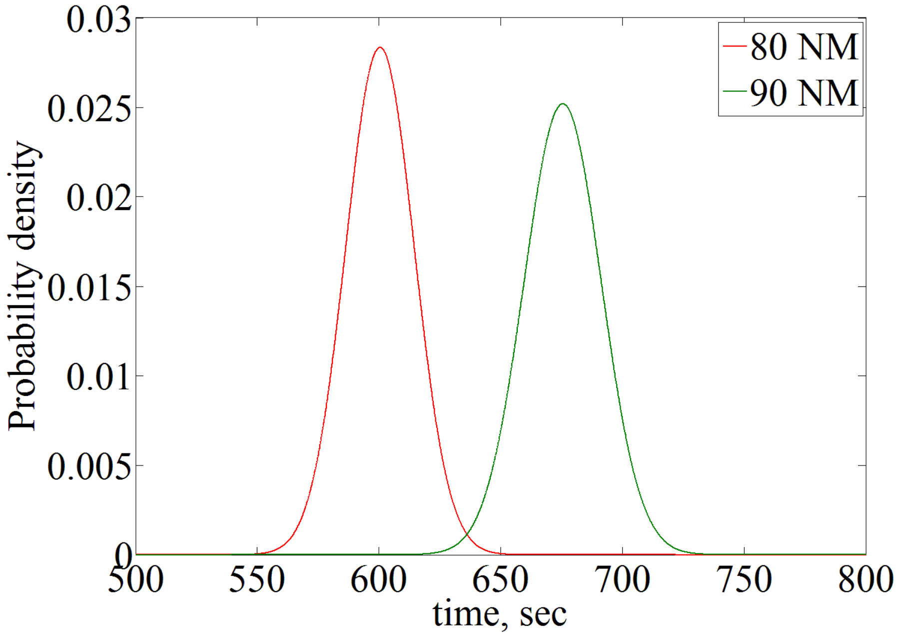


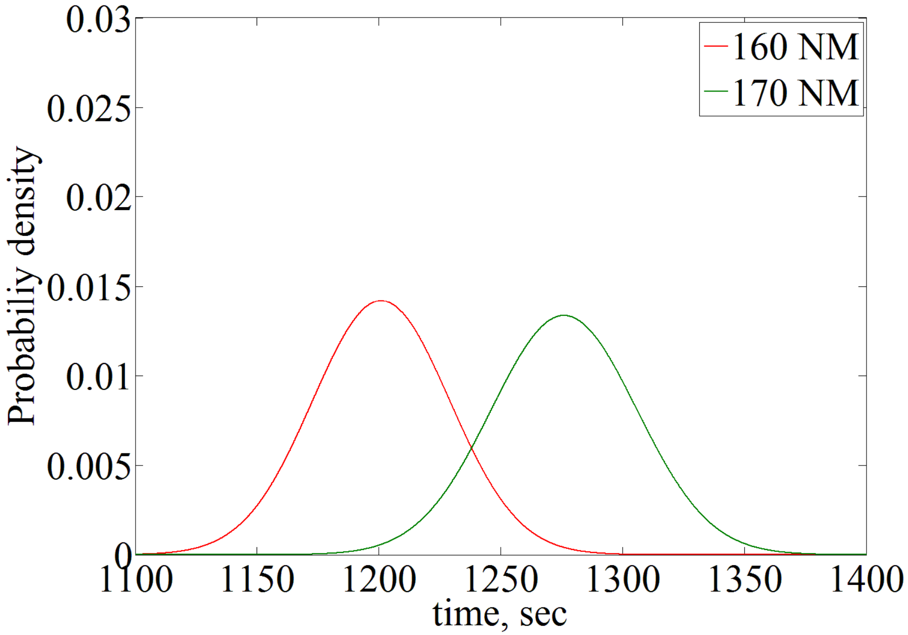
- 10 NM separation



Simulation Results

- 10 NM separation
  - X = 80 NM
  - X = 160 NM



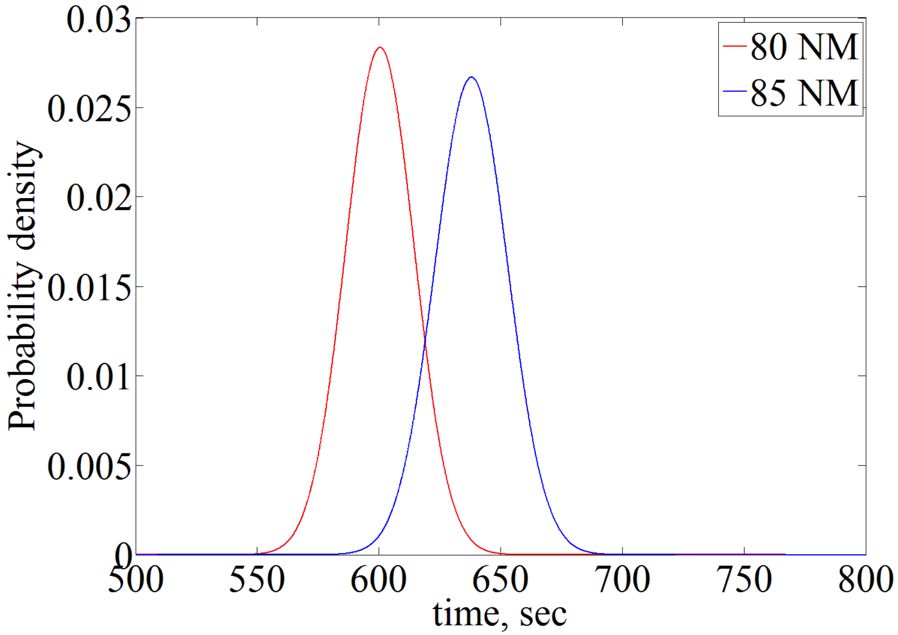


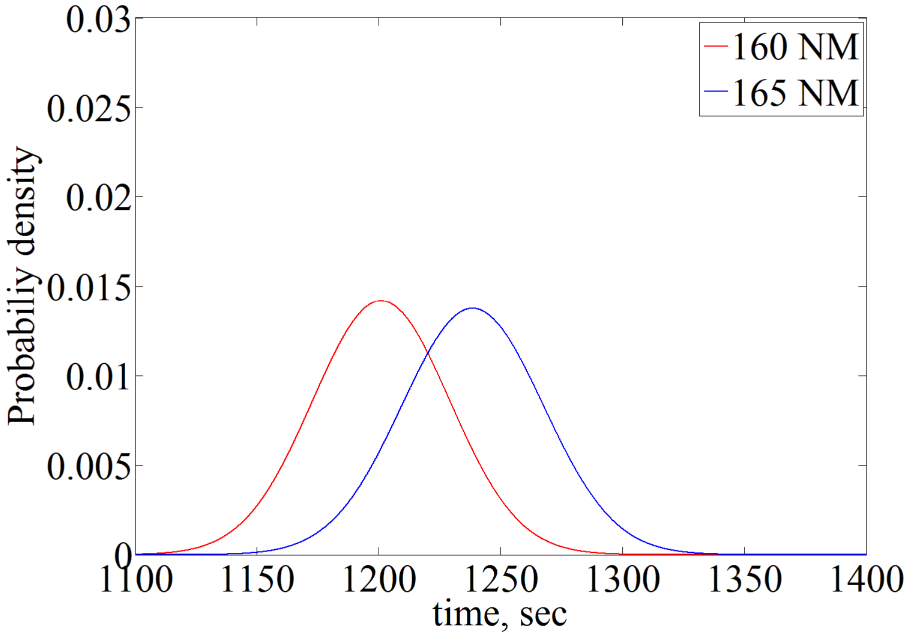
Case 1	Mean	Standard deviation
80 NM	600.49 sec.	14.07 sec.
90 NM	675.55 sec.	15.83 sec.

Case 1	Mean	Standard deviation
160 NM	1200.97 sec.	28.14 sec.
170 NM	1276.03 sec.	29.90 sec.

Simulation Results

- 5 NM separation
  - X = 80 NM
  - X = 160 NM



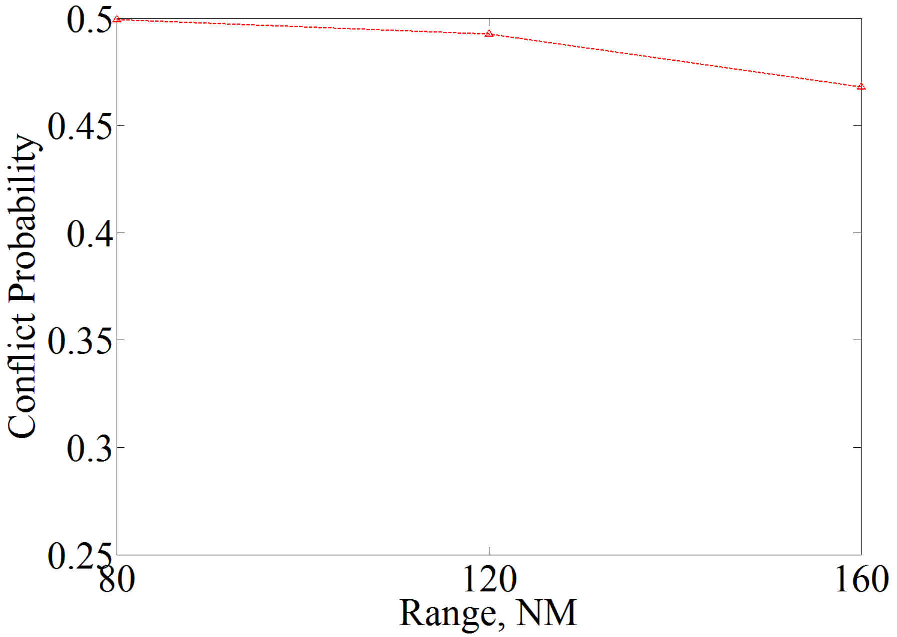


Case 1	Mean	Standard deviation
80 NM	600.49 sec.	14.07 sec.
85 NM	638.02 sec.	14.95 sec.

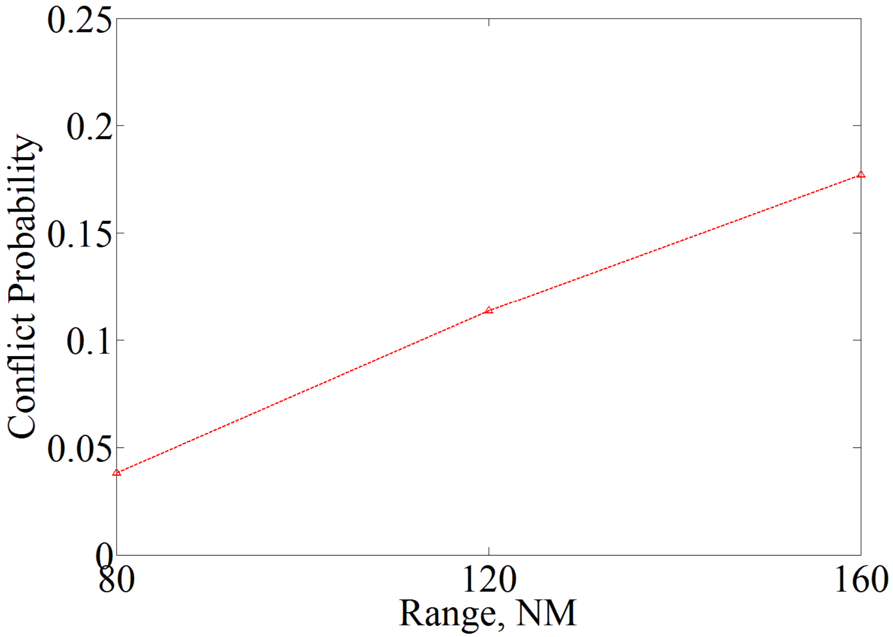
Case 1	Mean	Standard deviation
160 NM	1200.97 sec.	28.14 sec.
165 NM	1238.50 sec.	29.02 sec.

Simulation Results

- 5 NM separation

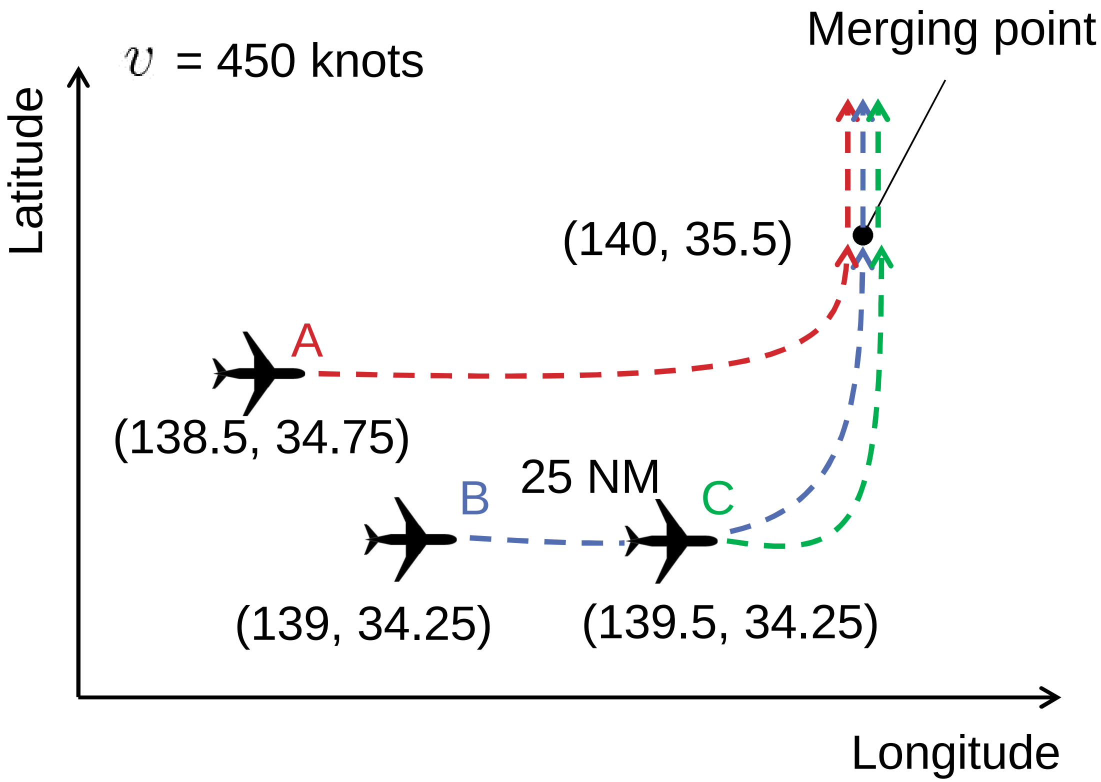


- 10 NM separation

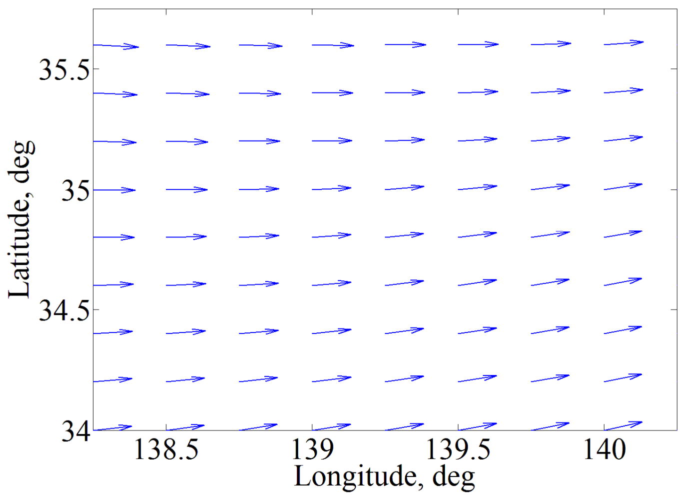


Simulation Results

- With meteorological predictions
  - Case 2

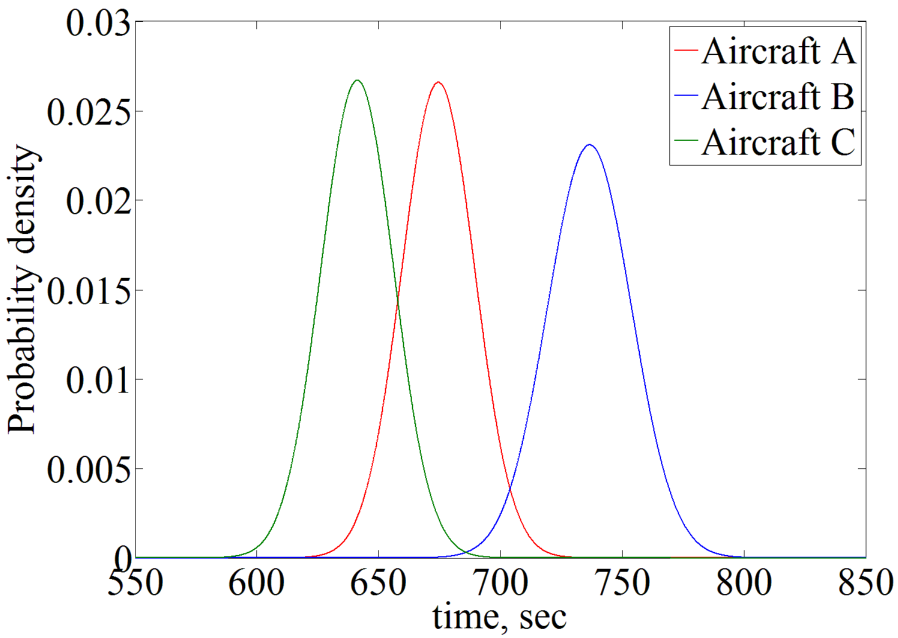


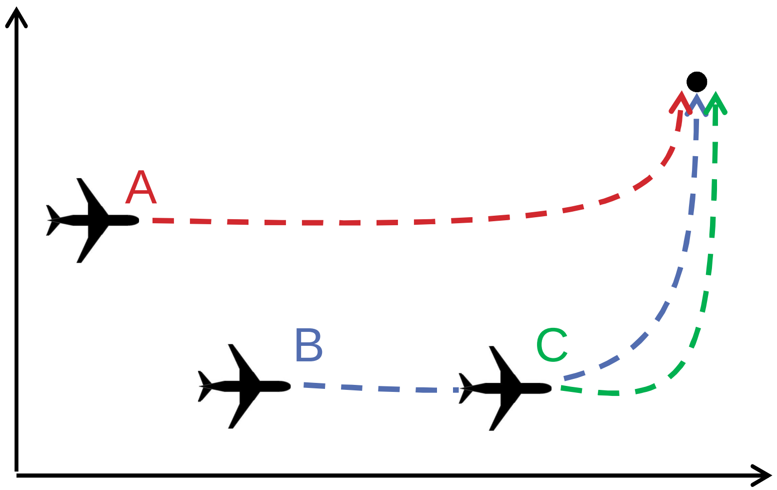
Predicted wind velocity



Simulation Results

- Case 2





Case 2	Mean	Standard deviation
Aircraft A	674.55 sec.	15.01 sec.
Aircraft B	736.66 sec.	17.27 sec.
Aircraft C	641.28 sec.	14.94 sec.

Case 2	Conflict Probability
A – B	0.16687
A – C	0.37491
B – C	0.0076496

Summary

- Conflict detection in the merging situations
  - Probabilistic approach
    - Consider various uncertainties
  - Generalized Polynomial Chaos
    - Easy to implement
    - Reduce computational burden
  - Conflict probability at merging point
    - Time of arrival at merging point
    - Expected value and standard deviation

Future Work

- 3 dimensional space
  - Descent phase
- Time-varying random variables
  - Increase the number of random variables
- Conflict resolution
  - Use the information of conflict probability
  - Change time of arrival