New trends in Air Traffic Complexity

D. Delahaye and S. Puechmorel

Applied Math Laboratory ENAC

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Agenda

- Why complexity metrics are needed ?

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- Automated Conflict Solver enhancement (robustness of the solution).

Complexity vs Workload

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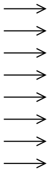
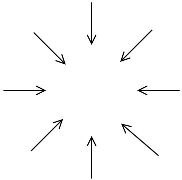
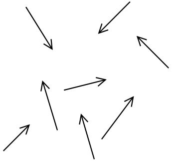
- Complexity is related to mixing behaviour.

Sensitivity-interdependance







No Sensitivity

No conflict

Easy situation

Hight sensitivity

Potential conflicts without interaction between solutions

Hight sensitivity

Potential conflicts with interactions between solutions

Hard situation

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- Dynamical system modeling of aircraft trajectories
- How a complexity map is built ?
- Results

Linear Dynamical System Modeling

The key idea is to model the set of aircraft trajectories by a linear dynamical system which is defined by the following equation :

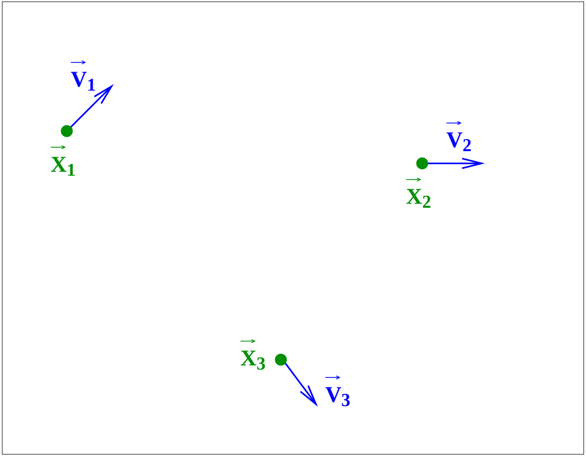
$$\dot{\vec{X}} = \mathbf{A} \cdot \vec{X} + \vec{B}$$

where $\vec{X}$ is the state vector of the system :

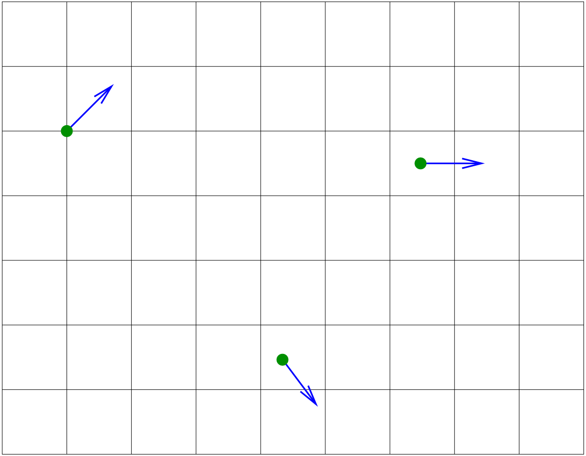
$$\vec{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Matrix $\mathbf{A}$ and vector $\vec{B}$ are the parameters of the model.

Regression of a Linear Dynamical System



Regression of a Linear Dynamical System



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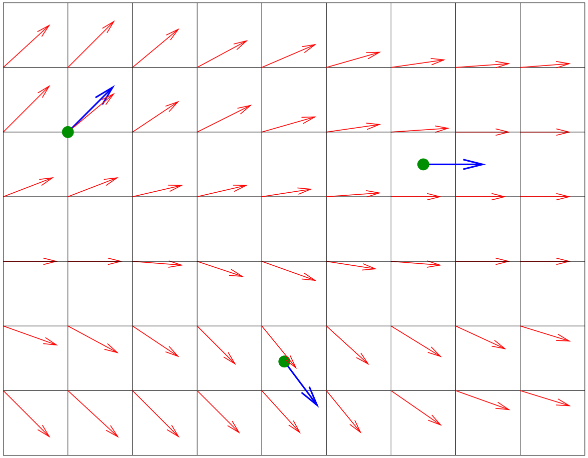

Regression of a Linear Dynamical System

- Based on a set of observations (positions and speeds), one has to find a dynamical system which fits those observations. Suppose that N obervations are given :

  Positions :

$$\vec{X}_i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}$$

  and speeds :

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  and speeds :
  $$\vec{V}_i = \begin{bmatrix} vx_i \\ vy_i \\ vz_i \end{bmatrix}$$
- A LMS precedure is applied in order to extract the matrix $\mathbf{A}$ and the vector $\vec{B}$.

Properties of the matrix $\mathbf{A}$

- When real part of the eigenvalues of matrix $\mathbf{A}$ is positive, the system is in expansion mode and when they are negative, the system is in contraction mode.

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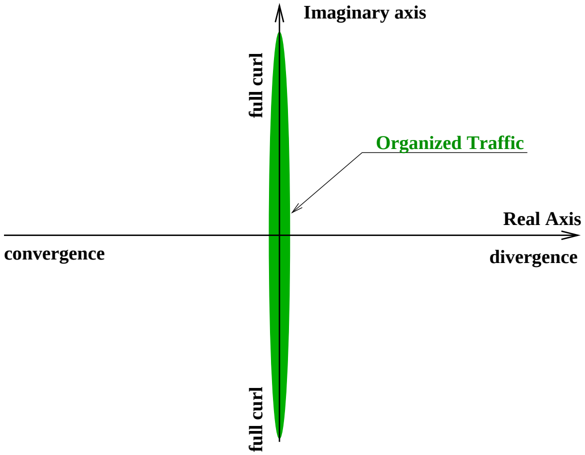
- When real part of the eigenvalues of matrix **A** is positive, the system is in expansion mode and when they are negative, the system is in contraction mode.
- Furthermore, the imaginary part of such eigenvalues are related with curl intensity of the field.

Linear Dynamical System Modeling : An example



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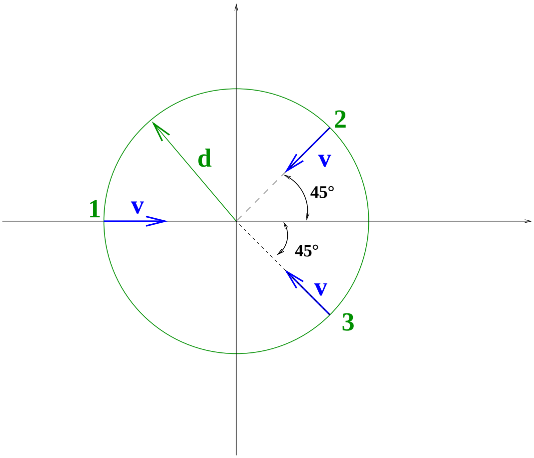

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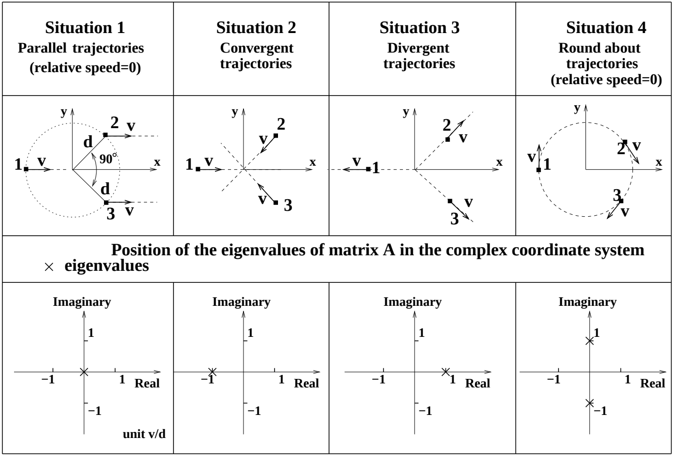

Linear Model limitations

- Give a global tendency of the traffic situation.

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- $\Rightarrow$ **Non Linear Extension**

Non Linear Extension in Space

$$\dot{\vec{X}} = \vec{f}(\vec{X})$$

Optimization problem

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$$minE = \sum_{i=1}^{i=N} \|\vec{V}_i - \vec{f}(\vec{X}_i)\|^2$$

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- and

$$min \int_{\mathbb{R}^3} \|\Delta \vec{f}(\vec{x})\|^2 d\vec{x} \ \text{ with } \ \Delta \vec{f} = \begin{bmatrix} \frac{\partial^2 f_x}{\partial x^2} + \frac{\partial^2 f_x}{\partial y^2} + \frac{\partial^2 f_x}{\partial z^2} \\ \frac{\partial^2 f_y}{\partial x^2} + \frac{\partial^2 f_y}{\partial y^2} + \frac{\partial^2 f_y}{\partial z^2} \\ \frac{\partial^2 f_z}{\partial x^2} + \frac{\partial^2 f_z}{\partial y^2} + \frac{\partial^2 f_z}{\partial z^2} \end{bmatrix}$$

Non Linear Extension in Space

Exact Solution (Amodei)

$$\vec{f}(\vec{X}) = \sum_{i=1}^{N} \mathbf{\Phi}(\|\vec{X} - \vec{X}_i\|).\vec{a}_i + \mathbf{A}.\vec{X} + \vec{B}$$

with

$$\mathbf{\Phi}(\|\vec{X} - \vec{X}_i\|) = \mathbf{Q}(\|\vec{X} - \vec{X}_i\|^3)$$

and

$$Q = \begin{bmatrix} \partial_{xx}^2 + \partial_{yy}^2 + \partial_{zz}^2 & 0 & 0 \\ 0 & \partial_{xx}^2 + \partial_{yy}^2 + \partial_{zz}^2 & 0 \\ 0 & 0 & \partial_{xx}^2 + \partial_{yy}^2 + \partial_{zz}^2 \end{bmatrix}$$

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$$min \int_{\mathbb{R}^3} \int_t \| \Delta \vec{f}(\vec{x}) \|^2 + \| \frac{\partial \vec{f}}{\partial t} \|^2 d\vec{x} dt$$

Non Linear Extension in Space and Time

Exact Solution (Puechmorel and Delahaye)

$$\vec{f}(\vec{X},t) = \sum_{i=1}^{N}\sum_{k=1}^{K} \mathbf{\Phi}(\|\vec{X}(t) - \vec{X}_i(t_k)\|, |t - t_k|).\vec{a}_{i,k} + \mathbf{A}.\vec{X} + \vec{B}$$

with

$$\mathbf{\Phi}(r,t) = \mathbf{diag}\left(\frac{\sigma}{r}.erf\left[\frac{r}{\sigma}.\frac{1}{\sqrt{2+\theta.|t|}}\right]\right)$$

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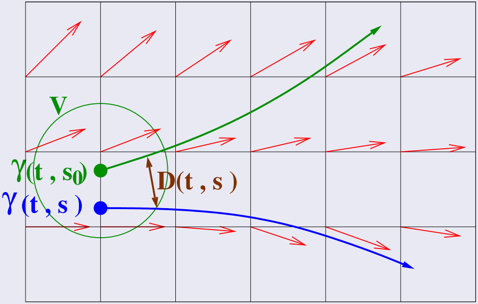
Characterization of sensitivity

Dynamical system trajectories



- Let $t \mapsto \gamma(t, s_0)$ be a nominal dynamical system trajectory (s_0 initial point)
- A perturbed trajectory is $t \mapsto \gamma(t, s)$ with $s \in V$ (V is an open neighborood of a given s_0).

Characterization of sensitivity

How fast two neighboring dynamical system trajectories diverge ?

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- One has to compute the evolution of the distance to nominal trajectory with respect to time : $D(t,s) = \|\gamma(t,s_0) - \gamma(t,s)\|$.

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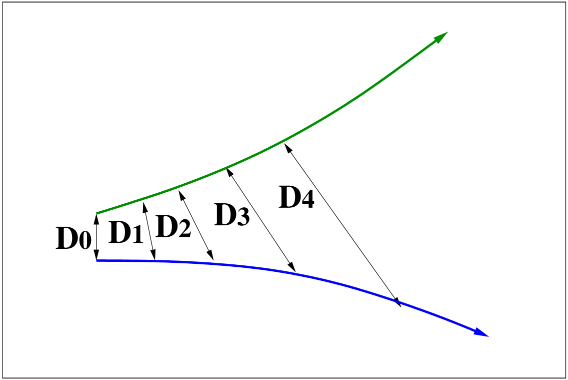
Computing $D(t, s)$

Main idea : when $t \mapsto \gamma(t, s)$ is the solution of a differential equation with initial condition $\gamma(0, s) = s$, it is possible to show that D itself satisfies a differential equation.

Local behaviour of trajectories



The variational equation

- $\gamma(t,s)$ being a flow :

$$\frac{\partial\gamma(t,s)}{\partial t} = F(t,\gamma(t,s)) \quad \gamma(0,s) = s$$

with F a smooth vector field.

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  with F a smooth vector field.

- then divergence of nearby trajectories can be found by solving :

$$\frac{\partial D(t,s)}{\partial t} = DF(t, \gamma(t, s_0)).D(t,s) \quad D(0,s) = \|s - s_0\|$$

  with DF the jacobian matrix of F (with respect to s).

The variational equation II

- If the three axis are considered simultaneously, the previous equation has the following matrix structure :

$$\frac{dM(t)}{dt} = DF(t, \gamma(t, s_0)).M(t) \quad M(0) = Id$$

This equation is called the variational equation of the flow.

Lyapunov exponents

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Lyapunov exponents

- Let $U^t(t)\Sigma(t)V(t) = M(t)$ be the SVD decomposition of $M(t)$.
- The Lyapunov exponents are limit values of the logarithms of the diagonal elements of $\Sigma(t)$.

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- Given an initial point, the Lyapunov exponents and the associated SVD decomposition provide us with a decomposition of space in principal directions and corresponding convergence/divergence rate.

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- It is a localized version of the complexity based on linear systems.

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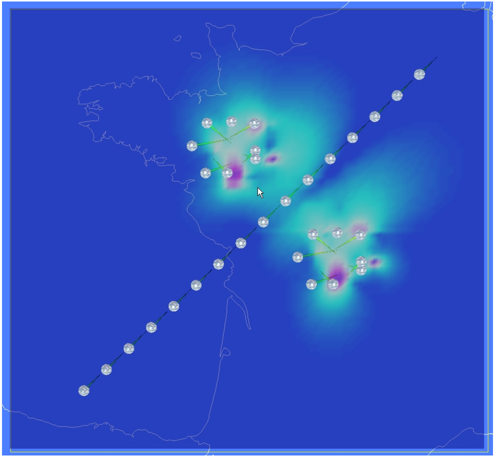

Results