2. Common Trends in Japanese- and European Airspace Data

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1. Introduction

Before an aircraft takes off, a lot of planning activity has already been done: Pilots of IFR flights (instrumental flight rule) were asked to submit their flight plan at least 2h prior to departure (EOBT). A flight plan contains a profile of the intended flight path which is then transmitted to the air traffic management center (ATMC) in Fukuoka and to the air traffic control facilities along the way of the aircraft. The role of flow management, which is one part of the ATMC, is to balance airspace demand with available capacity. A computer system monitors the airspace in real time, using flight plan, radar -and meteorological information. Based on this, air traffic management officers created safe and efficient traffic flows by coordinating with aircraft operators or by adjusting departure times of individual aircraft [1], [2]. Still prior to take-off, air-traffic control centers amended individual flight plans to create safe departure- and en-route conditions. Once an aircraft is in the air, it is guided by air-traffic controllers to its destination. In order to avoid collisions with other aircraft they may instruct the pilots to deviate the aircraft from their flight plans. Other reasons for deviations from flight plans are meteorological conditions, passenger delays, technical failures etc. This leads to gaps between the number of aircraft that are planned and the number that enters flight sectors in reality. Such gaps translate into safety problems and non-optimally used capacity. If the occurrence of such gaps would be known in advance, the performance of flow planning could be improved and controllers' workload could possibly be reduced.

Related work include the following: [3] and [4] identify binomial- and Poisson distributed counts of aircraft entering flight sectors. Moreover, [5] analyzed European Airspace data and found that gaps between the number of planned and realized entries into sectors occur systematically. In this paper we use the methodology of [5] to demonstrate that the same is true for the Japanese Airspace.

Gaps are differences between the number of planned and realized entries into a flight sector. They can be represented by three possible definitions

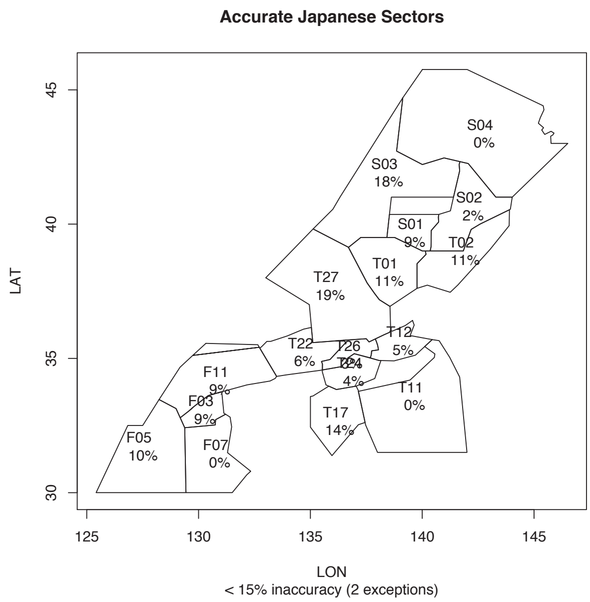

Figure 1: Selected sectors (simplified boundaries)

$$GAP_t = \begin{cases} REAL_t - PLN_t & \text{absolute} \\ REAL_t / PLN_t & \text{relative} \\ f(REAL_t, PLN_t, \mathbf{X}) & \text{functional} \end{cases}$$

where $REAL_t, t \in \mathbb{N}$ is the number of aircraft that entered a sector in time interval t and PLN_t is the number of aircraft that were supposed to enter it. Since the number of real entries into a flight sector is unknown, we consider GAP_t as a random process. All three definitions give insight in the phenomenon. For example absolute and relative gaps describe directly how the phenomenon appears to an observer. The functional definition models the joint distribution of $REAL$ and PLN. $\mathbf{X}$ is a vector describing the environment in which the variables are observed, for example the time of the day. It gives insight into how gaps are generated by the flow planning component.

In this paper we describe the univariate distributions P_{GAP_i} of all three processes. These are commonly referred to as marginal distributions. Unlike conditional distributions $P(X_t \mid X_{t-1}, X_{t-2}, ...)$, marginal distributions cannot completely specify a process [6]. Moreover, since observations $x_1, x_2, ..., x_n$ of a stochastic process are usually not independent, sampling

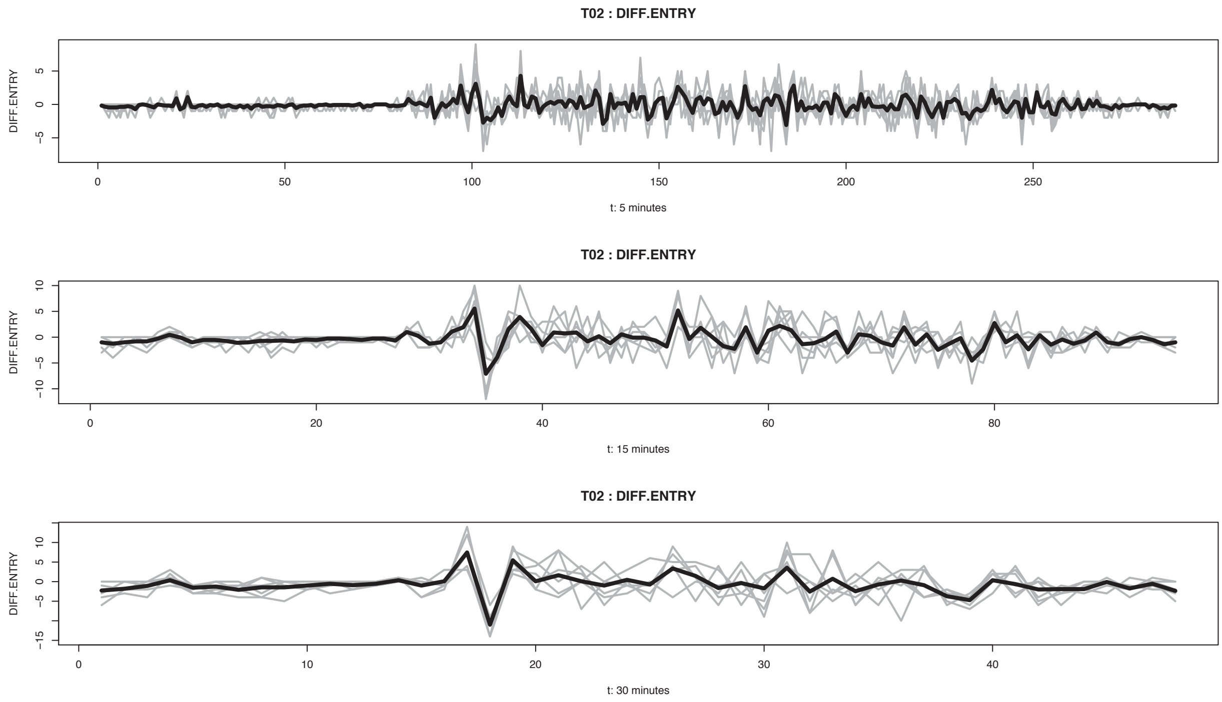

Figure 2: Time plots on different time-scales. Top: 5min, middle: 15 min, bottom: 30 min time scale.

its marginal distributions can be biased. On the other hand, marginal distributions summarize the global variation of a process. Sometimes they also provide insight in its dependency structure [7], [8]. When not stated otherwise, we report results from sampling marginal distributions of a single long realization of the stationary process GAP_t.

The paper is organized as follows: In the next section, the data sources are introduced. The following three sections show time-, distribution- and trend analysis respectively. A comparison with results from European Airspace can be found in section 6. Finally, interpretation and conclusions can be found in section 7.

2. Data source and selection

We use two data sources: the regulated flight plan data and the radar track data. These are trajectory based data formats. We assume a $climb - cruise - descent$ trajectory with constant climb - and descent phases and constant speed between two waypoints. Based on this, we aggregate the data on a sector level. All data is numerical. Radar data of the form (lat_t, lon_t, alt_t) is available in 10 sec intervals. Linear interpolations are made in order to determine crossing point - and time with the vertical planes of sector boundaries. We can expect accurate results in cruise phase. As far as the flight plan data is concerned, unreliable altitude information and other error sources lead to inexact entry time calculations into a flight sector [9]. An inspection showed that the average interval between two waypoints in cruise phase is about 6 min. Assuming that the true entry time into a sector takes place in the second half of the time between the last waypoint and the waypoint inside a sector, we can expect an average of 1.5 min entry time error per flight. These inaccuracies lead to time-distorted counts of the number of entries with respect to the true entry times.

For this study, we selected sectors where the crossing with the sector boundary of more than 85 % of entering aircraft could be determined. The selected sectors are 19 in total. In order to cover laterally the Japanese Airspace we added two more sectors (T27 and S03 with 81 % and 82 % of entry time accuracy respectively). The sectors belong to 3 control centers. 11 of them are from the Tokyo center, 4 from the Sapporo center, and 6 from the Fukuoka center. These sectors are generally en-route sectors. The sectors are shown in Figure 1. The numbers are the percentages of entering flights with inexact entry time calculations.

Flight plan and radar data is available for 2 working weeks in 2006 (we use Tuesday-Thursday as working week). The first week is August 22-24. The second week is November 14-16. In total, this

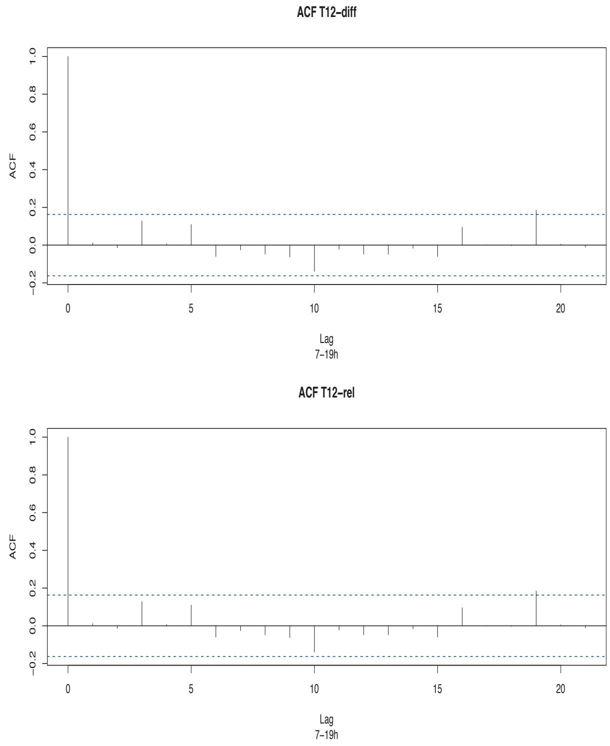

Figure 3: Sample autocorrelation. Top: PLN-REAL. bottom: PLN/REAL

gives 6 days of data. The daily average number of flights in both weeks is around 3500. A difference is in the daily average number of regulated flights: 996 in the summer, and 833 in the winter month. This is probably due to severe weather conditions in the August week (storm, rain), leading to decreased sector capacities.

In what follows, we present results from all 6 days of data. In a detailed report, we show that the main characteristics are the same in the summer- and winter data [10].

3. Time Plots

Figure 2 shows the process $GAP_t = PLN_t - REAL_t$ on three different time-scales. The timescales grow from 5 min (upper panel), over 15 min (middle) to 30 min (bottom). On the x-axes, the slot numbers are drawn. The upper panel has 288 slots (12 per hour), the middle one 96 (4 per hour) and the lower panel has 48 slots (2 per hour of the day). In each panel, 6 grey lines are superposed, one from each of the available days. The black line is the average over these 6 days. This representation of the time series suggests that successive days are independent observations of a same underlying process. When one accepts this assumption, standard statistical inferences (e.g. confidence intervals) can be made per time slot. In the 15- and 30 minutes slots, one can observe a certain regularity: strong peaks and valleys reproduce on a daily basis. For example at t = 9h (slot 36 and 18), a negative peak of value -10 can be seen. In the remaining time, the process fluctuates around an average value of 0. Indeed, a one-sided t-test agrees that this mean value is higher than a randomly selected one from the remaining process. The variance of this fluctuation looks constant during the day hours (7-19h).

Of all 21 sectors, strong visible peaks have been observed in three sectors: F05, T02 and T26. A more detailed analysis showed that in F05 and T02, a peak in planned traffic arrives with delay, causing the observed peak. As mentioned above, this can also be due to the inaccuracy in arrival time calculation. F05 has 10 % of inaccurate, and T02 11 % of inaccurate arrivals. In T26, a valley in planned traffic exists at t=9h (see [10]). The real traffic is smooth at this time, leading to the observed gaps. This might indeed be the effect of adjustment by ATC's.

Figure 3 shows the sample autocorrelation function (acf(k)) of the series $GAP_t = REAL_t - PLN_t$ (top) and $GAP_t = REAL_t/PLN_t$ (bottom)

$$acf(k) = \frac{\sum_{i=1}^{N-k}(x_i - \hat{\mu})(x_{i+k} - \hat{\mu})}{\sum_{i=1}^{N}(x_i - \hat{\mu})^2}, \quad 0 \leq k \leq 21$$

of one day (7-19h) until lag k=21, corresponding to roughly 2 hours. $\hat{\mu} = \frac{1}{N}\sum_{i=1}^{N} x_i$. The horizontal lines delimit the 95 % confidence interval for the autocorrelation of a process with independent and identically distributed variables. No meaningful significant coefficients appear in both plots. Thus, an analysis of the marginal distributions can be justified.

4. Marginal Distributions

The marginal distributions of the absolute, relative and functional processes have been analyzed.

Absolute Figure 4 shows a typical histogram of the marginal distribution of absolute gaps $GAP_t = REAL_t - PLN_t$ during the day (7-19h). Sector T12 has been selected for this example. It is a symmetric distribution. Its estimated mean and standard deviation are $\hat{\mu} = 0.21$ and $s = 2.6$ respectively. Superposed is a normal distribution with density

$$pr(x) = \frac{1}{s\sqrt{2\pi}} e^{-\frac{(x-\hat{\mu})^2}{2s^2}}$$

Sector	N	$\hat{\mu}$	s	$skew$	$kurt$	min	max
F03	870	0.2	2.3	0.0	3.3	-7	9
S02	870	-0.1	1.8	-0.2	3.3	-6	5
T11	870	-0.1	0.8	0.0	5.6	-3	4
T02	870	0.0	2.1	-0.2	3.8	-9	7
F21	870	-0.0	1.7	-0.1	3.2	-5	6
all		[-0.5, 1.1]	[0.8, 2.7]	[-0.4, 0.2]	[3.0, 5.6]	[-9, -3]	[4, 10]

Table 1: Marginal distribution of absolute gaps $GAP_t = REAL_t - PLN_t$ (7-19h). 5 randomly selected sectors. Last row: all 21 sectors.

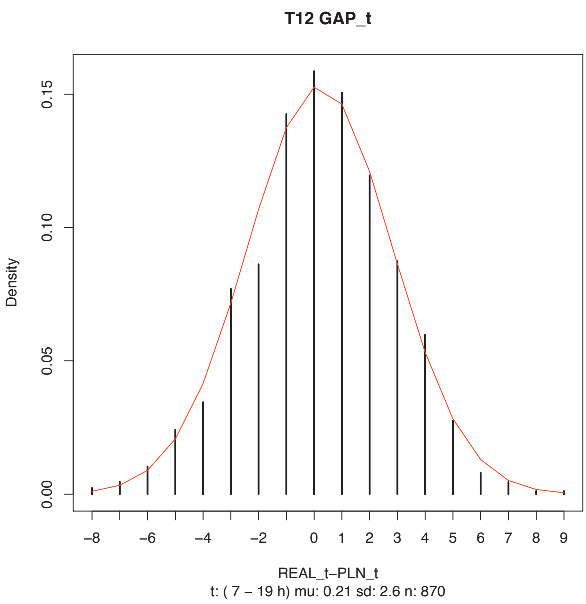

Figure 4: Histogram of absolute gaps $GAP_t = REAL_t - PLN_t$ (7-19h).

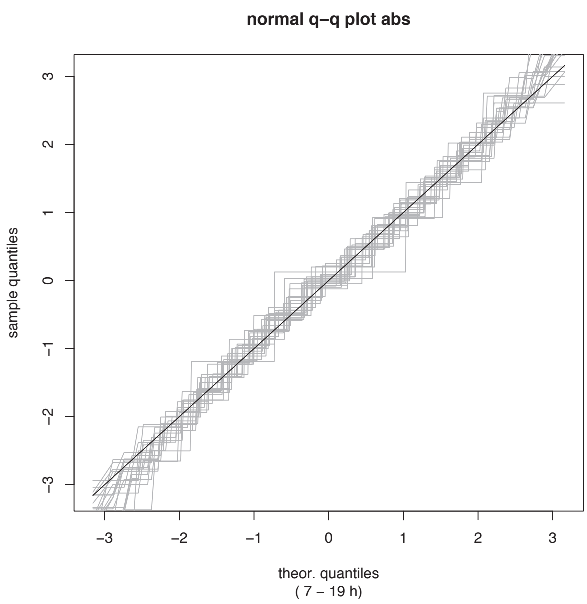

Figure 5: Quantile plots for absolute gaps of 21 sectors (7-19h).

Except from peaks at $GAP_t \in \{-1, 0, 1\}$ the distribution fits the data accurately. Table 1 shows descriptive statistics for five randomly selected sectors. They all have higher moments close to a Gaussian distribution: skewness (measure of symmetry) $\sim$ 0 and kurtosis (measure of 'peakedness') $\sim$ 3. However, note that the distribution looks slightly spikier than a Gaussian distribution. System-wide, the means of all 21 sectors lie in the range [-0.5, 1.1]. Their standard deviations lie in [0.8, 2.7], skewness in [-0.4, 0.2] and kurtosis in [3.0, 5.6] (last row). For an interpretation of the standard deviations please see section 6. According to the interquartile ranges, two outliers in the kurtosis could be identified. These were T11 (kurt=5.5) and T19 (kurt=4.4). Both indicate that the distributions are narrower than a Gaussian distribution. Visual inspection of these two distributions showed no other anomalies. Figure 5 plots the quantiles of the data against the quantiles of a Gaussian distribution (q-q plot). If the data were normally distributed, this plot would result in a straight line. The distributions of all 21 sectors are superposed. The data has been standardized, so the diagonal is the 45-degree line. The plot indicates no clear evidence against Gaussian distribution models.

Relative The upper part of Figure 6 shows the histogram of the marginal distribution of $GAP_t = REAL_t/PLN_t$ during the day (7-19h). It is antisymmetric to the right with $\hat{\mu} = 1.4$, $s = 1.2$ and skewness $skew = 2.3$. Outliers with a larger value than four times the standard deviation have been removed. Superposed is a log-normal distribution with density

$$pr(x) = \frac{1}{sx\sqrt{2\pi}} e^{-\frac{(ln\ x - \hat{\mu})^2}{2s^2}}$$

The lower part shows their logarithm $GAP_t = log(REAL_t/PLN_t)$. It still looks slightly antisymmetric to the right. It has mean $\hat{\mu} = 0.1$, standard deviation $s = 0.9$ and skewness $skew = 0.2$. However, a Gaussian distribution is superposed. It

Sector	N	$\hat{\mu}$	s	skew	kurt	min	max
T22	870	-0.1	0.8	0.0	3.8	-2.8	2.6
F07	870	-0.2	0.9	-0.1	3.1	-2.7	2.6
T26	870	0.3	0.8	0.1	3.9	-2.4	3.0
T02	870	0.0	1.0	0.0	3.1	-2.9	2.7
T24	870	0.2	0.8	0.2	3.8	-2.8	2.8
all		[-0.2, 0.3]	[0.7, 1.0]	[-0.1, 0.3]	[2.4, 3.9]	[-2.9, -1.9]	[2.2, 3.0]

Table 2: Marginal distribution of relative gaps $GAP_t = log(REAL_t/PLN_t)$ (7-19h) on 5 randomly selected sectors. Last row: all 21 sectors.

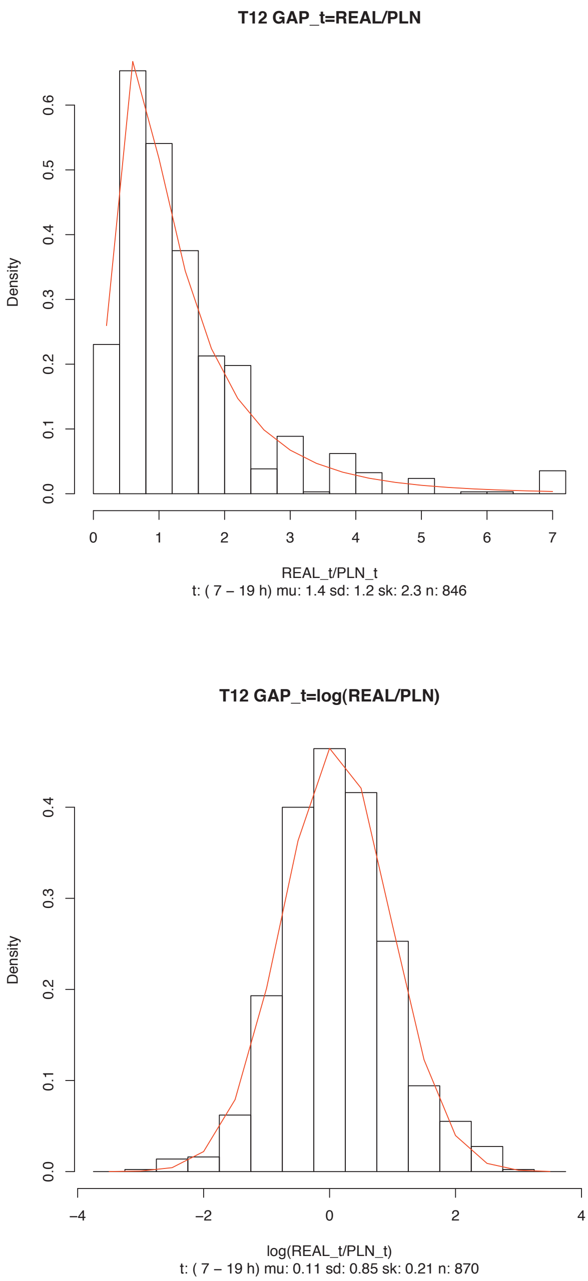

Figure 6: Histograms of relative gaps $GAP_t = REAL_t/PLN_t$ (top) and of their logarithm (bottom).

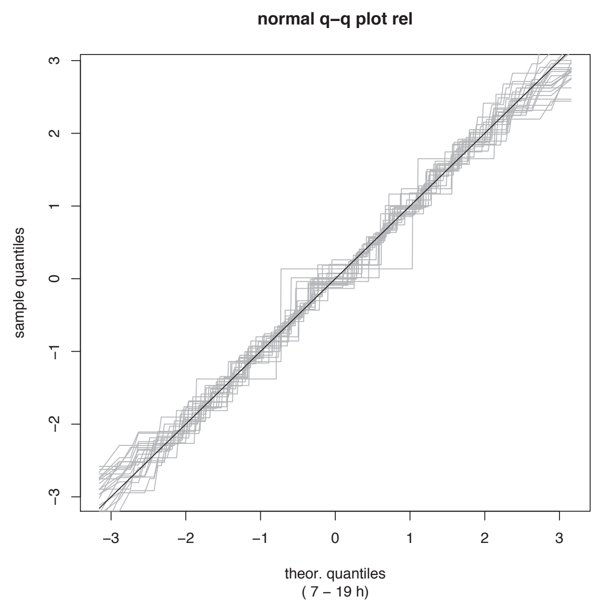

Figure 7: Quantile plots for relative gaps of 21 sectors (7-19h).

fits the data reasonably. Table 2 shows descriptive statistics for this representation of relative gaps. No obvious exceptions from Gaussian distributions can be seen. In all 21 sectors, skewness lies in the interval [-0.1, 0.3] and kurtosis in [2.4, 3.9] (last row). The q-q plot for all 21 sectors can be seen in Figure 7. No strong deviations from Gaussian distributions can be observed, although the tails show slightly higher probabilities than Gaussian distributions.

Functional Figure 8 shows the histogram of the variable $REAL_t$ of the sector T01, conditioned on the planned traffic PLN_t: $P(REAL_t|PLN_t)$. In the example $PLN_t = 2$, which is the average traffic density for this sector. The distribution is right skewed. The variable is positive (including 0) and discrete. Superposed are Poisson (green, dotted) and binomial distributions (red). Both distributions are accepted by a χ^2 goodness-of-fit test (on a 5 % level)

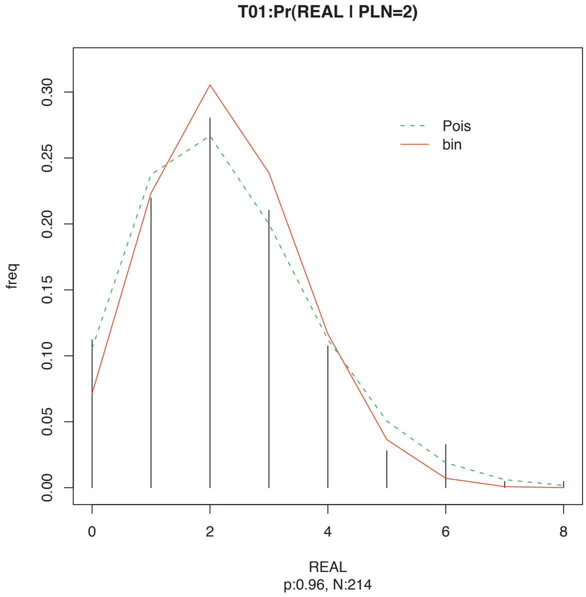

Figure 8: Number of real entries ($REAL_t$) conditioned on on number of planned entries ($PLN_t = 2$).

Distribution	Cond.	χ^2
Poisson	PLN	70 %
Binomial	PLN	26 %
Poisson	$PLN \leq 2$	80 %
Poisson	$PLN > 2$	55 %

Table 3: Goodness-of-fit of Poisson and Binomial distributions.

A system-wide comparison is summarized in table 3. We selected randomly 100 sectors and conditions and evaluated the goodness-of-fit of a Poisson and of a binomial distribution with a χ^2 test. Globally, 70 % of the distributions can be seen as Poisson and 26 % as binomial. When planned traffic $PLN \leq 2$, Poisson distributions are accepted in 80 % of the cases.

5. Trends

Figure 9 shows the scatterplot of PLN_t against $REAL_t$ on a 5 min timescale. The range of both variables is around [0,8]. Since there are $N = 870$ points in the sample, we visualize the distribution of the cloud by the background color (light: few values, dim: many values). One can see a single peak around (1,2) and that the cloud is distributed symmetrically around this peak. Negative values seem to be 'cut-off'.

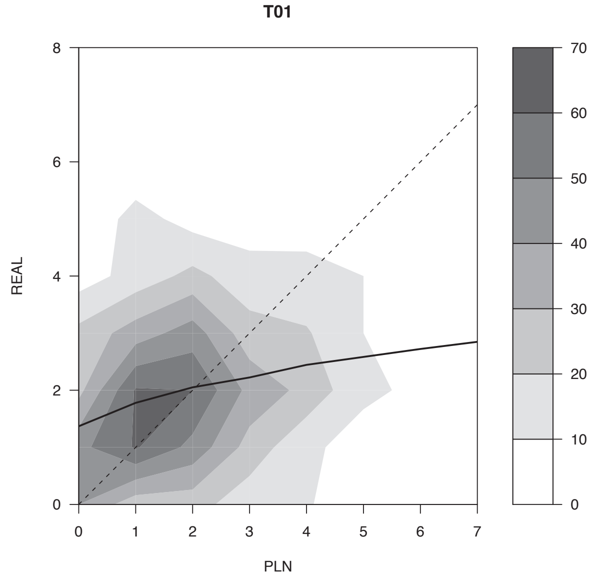

Figure 9: Relationship between number of planned arrivals PLN (x-axis) and average number of real arrivals $REAL$ (y axis). Distribution and sample means (bold line).

The sample conditional mean is

$$\hat{\mu}(REAL_t|PLN_t = k) = \frac{1}{n_k} \sum_{PLN_i=k} real_i$$

where n_k is the number of observations with $PLN_t = k$. As a function of k, it has a logarithm-like shape (bold line). In particular at $PLN_t = 0$, the mean is > 0.

Three simple descriptions of this relationship are $f(x) \in \{log(x), \sqrt{x}, 1 - 1/x\}$. A common technique to analyze count-variables is to model the logarithm of their mean [11]. This transformation assures non-negative mean values, which is a constraint for certain probability distributions (but it affects the interpretation of parameters). Since the variable $REAL_t$ is a count, we assume

$$log(\mu(REAL_t \mid PLN_t) = \alpha f(PLN_t) + \beta$$

with $\alpha, \beta, \mu(REAL_t \mid PLN_t) > 0 \in \mathbb{R}$. We fit the above models by maximum likelihood, assuming that $P(REAL_t \mid PLN_t)$ follows a Poisson distribution with mean $e^{\mu(REAL_t \mid PLN_t)}$. Note that the model using the logarithmic trend can also be written as the power of α. The predicted mean values of the three models are superposed in Figure 10. They all describe evenly well the main shape of the sample means.

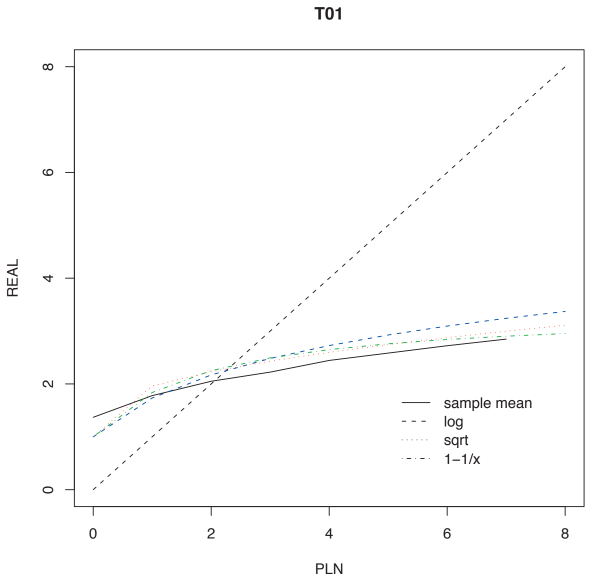

Figure 10: Candidate trend functions. x-axis: number of planned arrivals, y axis: average number of real arrivals.

	Europe	Japan
s	[1.4, 2.3]	[0.8, 2.7]
tails	differ	OK
tails(log)	differ	OK

Table 4: Comparison of European and Japanese Airspace

6. Comparison with European Airspace

A similar analysis has been conducted with data from Central European Airspace [5]. The main characteristics — shape of processes, marginal distributions and trends — are the same in both airspaces. There are only minor differences, which are summarized in Table 4.

The variation of the gaps (s = standard deviation) in Europe lies in the interval [1.4, 2.3] and in Japan in [0.8, 2.7]. Before comparing directly, one can argue that this variation depends on the traffic densities. An analysis showed that the variation grows with the average traffic densities, as expected. A normalization by the logarithm of the average traffic density leads to near-constant variability for the Japanese- and the European sectors. Details of this effect are currently under investigation. The next two rows concern the marginal distributions. Another study showed that in European Airspace, the tail probabilities of normal and log-normal distributions are lower and than the empirical probabilities [5]. This might be the effect of non-heterogeneous behavior, for example different take-off strategies of pilots. According to our study, this tail pattern cannot be seen in the Japanese Airspace data.

7. Interpretation

Uncertainty We analyzed three definitions of gaps between the number of planned and realized entries in a sector. We observed that the patterns of planned and realized traffic are similar and that they repeat on a daily basis. Moreover, absolute $PLN_t - REAL_t$ and relative $PLN_t/REAL_t$ gaps fluctuate around 0 and 1 and have constant variance during the day and during the night. This suggests that the phenomenon of gaps is time-invariant (stationary during the day (7-19h)). In particular, no peak hours of gaps exist. We then analyzed the marginal distributions of the three definitions of gaps during the day (7-19h).

a. $REAL_t - PLN_t$ (absolute definition)
Absolute gaps are symmetrically distributed with $\mu \sim 0$ and $sd \sim 2$. This means that roughly the same number of aircraft than planned arrives at sector entries. It also means that in 96 % of the cases, ± 4 of the planned aircraft arrive. Except two outliers in the kurtosis, no strong disagreement with Gaussian distributions in the core and the tail could be identified. Gaussian distributions can be expected when uncertainty factors are independent from each other.

b. $REAL_t/PLN_t$ (relative definition)
Relative gaps are distributed asymmetrically around $\mu \sim 1$. One reason is that the variable is positive by definition and that the mean is close to 0, so it is ‘cut-off’ on the negative values. The distributions of their logarithms show no strong disagreement with Gaussian distributions, although their tail probabilities are slightly too high and their logarithms seem slightly right-skewed. As a consequence, log-normal distributions are candidates to describe relative gaps. Log-normal distributions occur when the uncertainty factors are independent but with multiplicative effect.

c. $REAL_t = f(PLN_t, t)$ (functional definition)
Gaps as a function of planned traffic are distributed asymmetrically. Poisson distributions accurately characterize this representation of gaps. This gives an idea of the variation of the number of arrivals in a sector because mean and variance are the same for a Poisson distribution. The Poisson distribution can be derived analytically from counting the number of events in larger time intervals when the probability of occurrence of events are

constant and independent over time, characterizing ‘total randomness’. Moreover, Poisson distributions are the limiting distributions when several Point processes are superposed [12]. But one cannot deduce from an observed Poisson distribution that the probability of events is constant and independent [13]. Moreover distributions with similar shape exist (e.g. negative binomial distribution), and separation constraints contradict with exponentially distributed inter-arrival times. To conclude, it is not enough information to unambiguously draw conclusions about the underlying mechanism of the phenomenon [13].

Trends We observed that the average of real arrivals decreases with large number of planned arrivals. More specifically, systematic under-deliveries have to be expected during high traffic densities. This is counter-intuitive since one expects that different uncertainty factors cancel out in average. The phenomenon can be partly attributed to random disturbances of the flight schedules [14]. This information is useful for flow planning: avoid sequences of high planned densities to allow for compensation of uncertainties.

Similarities with European Airspace We have seen that the main characteristics — shape of processes, marginal distributions and trends — are the same in both airspaces. This is expected since the physical constraints of both systems are similar.

8. Conclusion

We analyzed data from the Japanese Airspace on a sector level. We compared the number of aircraft that were planned to enter sectors (flight plan data) with the number that entered them in reality (radar data). We also compared our findings with results from European Airspace. Our hypothesis was that due to operational uncertainties (e.g. delays, technical failures, etc.), there are systematic gaps between these two numbers. From our analysis we conclude that in both, European and Japanese Airspace, (i) gaps between the number of planned and realized traffic occur systematically, (ii) under-delivery increases with higher traffic densities and (iii) the size of the gaps can be described with Poisson distributions. This is counter-intuitive since one expects that different uncertainty factors cancel out in average. Such information is useful for flow planning: avoid sequences of high planned densities to allow for compensation of uncertainties. These findings are empirical. We found an accurate description of the data and its variation. A part of this phenomenon can be attributed to complete random disturbances of flight schedules. In the future, an analysis of the underlying mechanisms is needed to derive strategies to improve the performance of flow management.

References

[1] *Aeronautical Information Manual Japan. Vol 14*. Japan Aircraft Pilot Association, Tokyo. Japan, 2008.

[2] *Aeronautical Information Publication (AIP). ENR-1.12*. Japan Civil Aviation Bureau. Ministry of Land, Infrastructure, Transport and Tourism, Tokyo. Japan, 2008.

[3] C. Wanke, M.B. Callaham, D.P. Greenbaum, and A.J. Masalonis. Measuring uncertainty in airspace demand predictions for traffic flow management applications. *AIAA Guidance, Navigation and Control Conference. Austin, TX, USA.*, 2003.

[4] C. Wanke, L. Song, S. Zobell, D. Greenbaum, and S. Mulgund. Probabilistic congestion management. *Proceedings of the 6th Europe-USA ATM Seminar. Baltimore. US.*, 2005.

[5] C. Gwiggner. Analyse des incertitudes dans les flux du trafic aérien. *PhD Thesis. Ecole Polytechnique*, 2007.

[6] J.K. Lindsey. *Statistical Analysis of Stochastic Processes in Time*. Cambridge University Press, 2004.

[7] F. Baccelli. Internet: Modéliser le trafic pour mieux le gérer. In Société de mathématique de France (SMF) et Societé de mathématiques appliquées et industrielles (SMAI), editor, *l'Explosion des mathématiques*, pages 75–79. Ecole polytechnique, Palaiseau, 2002.

[8] O. Cappé, E. Moulines, J-C. Pesquet, A. Petropulu, and Z. Yang. Long-range dependence and heavy-tail modeling for teletraffic data. *IEEE Signal Processing Magazine*, 19(3):14–27, 2002.

[9] C. Gwiggner. Accuracy of regulated flight plan data. Technical report, ENRI, 2008.

[10] C. Gwiggner. Common Trends in Japanese- and European Airspace Data — Complete Report. Technical report, ENRI, 2008.

[11] J.K. Lindsey. *Modelling Frequency and Count Data*. Oxford University Press, 1995.

[12] D.R. Cox and V. Isham. *Point Processes*. Chapman and Hall, 1980.

[13] D.J. Daley and D. Vere-Jones. *An Introduction to the theory of Point Processes*. Springer, 1988.

[14] C. Gwiggner. Consequences of independence assumptions in ATFM. *Submitted to International Journal of Production Research (IJPR)*, 2007.